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CREATIVE DESTRUCTION, DISTANCE TO FRONTIER, AND ECONOMIC DEVELOPMENT

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Abstract

We construct a model of creative destruction with endogenous firm dynamics. We integrate the theory into a general equilibrium multi-country model of technological convergence where countries interact via international spillovers. We derive implications for both firm dynamics and aggregate productivity dynamics. In richer economies, firms are on average larger and the best firms grow larger over time. In poorer economies, there is little creative destruction, low selection, and firms remain small. We estimate the parameters of the model using firm-level data for India and the United States. We study the effect of counterfactual policy reforms. Industrial policy that selectively targets the more productive firms can be beneficial in poor countries while being harmful in countries close to the economic frontier. The findings echo Acemoglu et al. (2006).

1 Introduction

Since the seminal work by Aghion and Howitt (1992) the process of creative destruction has been inextricably linked to economic growth and rising living standards. Its broad spectrum policy implications cover areas such as patent protection and competition policy. This debate, however, has often focused on developed economies. Is creative destruction as important for developing economies?

Acemoglu et al. (2006) argue that developing economies typically rely less on innovation and creative destruction and instead generate most of their growth through the imitation and adoption of technologies already used by firms in more advanced countries. As the process of convergence proceeds, the potential for imitation and technology adoption wanes and firms must switch on the engine of innovation and selection of the best firms in order to continue their process of convergence to the technology frontier. Yet, this path is full of perils. Countries that fail to make their transition into innovation-led growth stop converging. This observation also implies that different policies and institutions may be appropriate (or optimal) at different stages of development. In particular, for countries far away from the technology frontier, economic growth may be aided by government interventions which selectively support specific firms, industries, and geographical areas. These discriminatory policies often come at the detriment of competition, as they impose implicit or explicit barriers on the entry and growth of new firms. In their view, in an environment characterized by large wedges and market failures, the benefits of overcoming credit market and contractual frictions outweigh the cost of reducing creative destruction. However, as an economy approaches the world technology frontier, policies that choke off competition and selection eventually become a burden on further development. As innovation becomes the main source of convergence, countries must introduce economic reforms to liberalize entry and foster creative destruction. Promoting human capital and managerial selection is also crucial for the long-run performance of the country.

Zilibotti (2017) presents empirical evidence from a panel of 43 non-OECD economies in the period 1965–2014 which is consistent with the prediction of the theory. He shows that barriers to entry are relatively less important (and can in some instances even be beneficial) in countries that are very far from the technology frontier. However, as countries come closer to the technology frontier, their growth rate declines faster than in countries promoting creative destruction through low barriers to entry. He also documents that convergence declines over time at a slower rate in countries investing a larger share of their GDP in R&D—an indication of the growing importance of innovation as economies approach the technology frontier.

Vandenbussche et al. (2006) document a similar pattern for human capital. They show that the share of tertiary education in the labor force becomes increasingly important to sustain technological convergence as countries approach the technology frontier. Arguably, this share is a proxy for the quality of managers and entrepreneurs in an economy. These distinct pieces of empirical evidence corroborate the view that while countries at an early stage of economic growth can converge fast to the technology frontier even with little creative destruction, how far the convergence process can go in the long run hinges on the quality, selection, and innovation potential of their firms.

In this chapter, we propose a theory consistent with the view that the relative importance of technology adoption versus creative destruction changes over the process of economic development. The novelty of our theory is its focus on firm dynamics, paving the way to the possibility of estimating and testing its prediction using firm-level data rather than aggregate data.

Building Blocks. The theory has three building blocks. The first is the celebrated Schumpeterian model of firm dynamics with creative destruction by Klette and Kortum (2004a). This model provides a tight link between creative destruction, the process of firm-dynamics, and the resulting firm size distribution. Most importantly, it enables us to infer cross-country differences in creative destruction from differences in the size distribution of firms.

The second building block is the idea that firms are heterogeneous in their growth potential, namely, some firms are intrinsically more dynamic than others. We closely follow the analysis of Akcigit et al. (2021), who argue that such heterogeneity is particularly important to understand firm dynamics in developing countries and estimate a related model to firm-level data from India and the US.[1] Like them we assume that some entrepreneurs are "transformative" and have the necessary skills to expand, whereas "subsistence entrepreneurs" may simply never grow independently of the environment they operate in.[2] The existence of different firm types highlights the role of creative destruction for the process of selection: if creative destruction is low, subsistence firms are only replaced slowly and in equilibrium a large share of the economy's resources are allocated to firms that do not grow.

The third building block is a process of knowledge diffusion inspired by Acemoglu et al. (2006): countries further away from the technological frontier can benefit from technology adoption as a substitute for low creative destruction. This ingredient generates a link between cross-country differences in creative destruction and aggregate productivity. In particular, the theory predicts that countries that are far from the technology frontier can grow fast even if firms are not very innovative. However, in the long run the steady-state distance from the technology frontier at which each country settles hinges on the rate of creative destruction in each country. In turn, this is determined by the quality (e.g., entrepreneurial and managerial human capital) of incumbent and entrant firms and by the extent to which barriers to entry limit churning and creative destruction. These characteristics ultimately pin down the ranking of nations in the stationary productivity distribution of the global economy.

Mechanism of the Theory. As in Klette and Kortum (2004a), in our model productivity growth stems from both the entry of new firms and the expansion of existing firms. An incumbent

[1] A similar structure is also considered in Acemoglu et al. (2018).

[2] Empirical evidence for this dichotomy is presented in Schoar (2010) and Decker et al. (2014). In the context of developing countries, Banerjee et al. (2015) and De Mel et al. (2008) stress the importance of persistent differences in growth potential.

firm is indexed by the measure of products for which it owns the best technology. Transformative firms grow by taking over other firms' products and shrink because of creative destruction through both the entry of new firms and the expansion (at their expense) of other incumbent firms. Subsistence firms are by construction stagnant and simply produce in the product market they initially entered in until they get replaced. In our theory, creative destruction is both a cause and consequence of development. On the one hand, creative destruction is a necessary ingredient for selection and for subsistence firms to exit. In doing so, it reallocates resources to transformative entrepreneurs who can use such resources more productively. On the other hand, if the majority of entering firms are subsistence producers, creative destruction is bound to be low, as there are simply too few transformative firms to meaningfully affect the process of selection at the aggregate level. Creative destruction, the survival chances of subsistence firms and the firm size distribution are thus all jointly determined in equilibrium and linked to the level of aggregate productivity.

Estimation. We estimate our model using firm-level data from the US and India. We assume the US and India differ in three dimensions: the share of transformative firms, the innovation efficiency of transformative firms and the cost of entry. We think of these parameters as reduced-form stand-ins for a variety of institutional and technological differences between rich and poor countries. The share of transformative firms in India could for example be lower than in the US if entrepreneurial human capital is relatively scarce or frictions on the labor market reduce the opportunity costs of entrepreneurship for firms with little growth potential. Entry costs could be higher if bureaucratic red-tape or other license requirements are more onerous in India. Finally, the expansion incentives for transformative firms could be lower if credit market frictions are more severe or other forms of size-dependent policies reduce the incentives to grow large - see Akcigit et al. (2021).

We show how one can estimate these three parameters directly from three moments of the firm-size distribution: the entry rate, the extent of life-cycle growth and the share of small firms. Our estimation yields three important findings. First, the Indian economy is disadvantaged along all three dimensions: it has higher entry costs, lower incumbent innovation efficiency and a larger share of subsistence firms. Second, the rate of creative destruction implied by our estimates is much lower in India relative to the US. Hence, as hypothesized by Acemoglu et al. (2006), the firm-level data exactly shows that a relatively backward economy like India has lower creative destruction. Finally, we find that the differences in innovation efficiency (that are related to the human capital of local entrepreneurs) are much larger than those in entry costs. Consequently, the main reason why the US has higher creative destruction is not that entry is easier but that existing firms innovate and expand at a much faster rate.[3]

Because we can derive closed-form expressions for our calibration targets, our theory concisely highlights which moments in the raw data inform these conclusions. Empirically, Indian firms are (compared to the US) small and they experience little life-cycle growth. At the same time, entry and exit rates are quite similar. The fact that exit rates are similar, despite the fact that firms are small and hence close to exit threshold, implies that creative destruction in India has to be small. Furthermore, the absence of life-cycle firm growth in India paired with the abundance of very small firms implies that transformative firms in India are not very efficient to expand and that there are not many of them. Finally, to rationalize a common entry rate across countries despite the fact that entrants in the US face tougher competition from transformative firms implies that entry costs have to also be smaller in the US.

Policy. We then use our calibrated model to gauge the effectiveness of growth policy in India. We are particularly interested in whether policies that protect firms from competition might be beneficial for innovation and aggregate productivity. When all firms are ex-ante identical, restricting entry necessarily reduces creative destruction and productivity growth. However, when entrepreneurial talent is scarce, there is endogenous selection: large firms are on average of better quality and have a higher growth potential than small firms and the average entrant. This raises a case for protecting and supporting dynamic firms, especially when there is only a small number of them. Easy entry not only replaces good firms with worse firms but in addition also deters innovation investments from dynamic incumbents out of fear of business stealing. In this scenario, a poor economy has low productivity growth for two reasons. First, most firms are of low quality and are intrinsically stagnant. Second, the few good entrepreneurs are discouraged by the threat of replacement. This argument has two implications, one positive and one normative, that our stylized model spells out. On the one hand, in less developed countries there is low creative destruction and firms remain on average small. On the other hand, a benevolent government that can identify the good firms could wish to protect them from replacement at the hand of other (likely worse) firms.

Thus, little creative destruction is in part a symptom of underdevelopment: there are few dynamic firms around. In some circumstances, promoting competition and free entry may not be the optimal policy: it can reduce dynamic selection and the growth and life span of the few

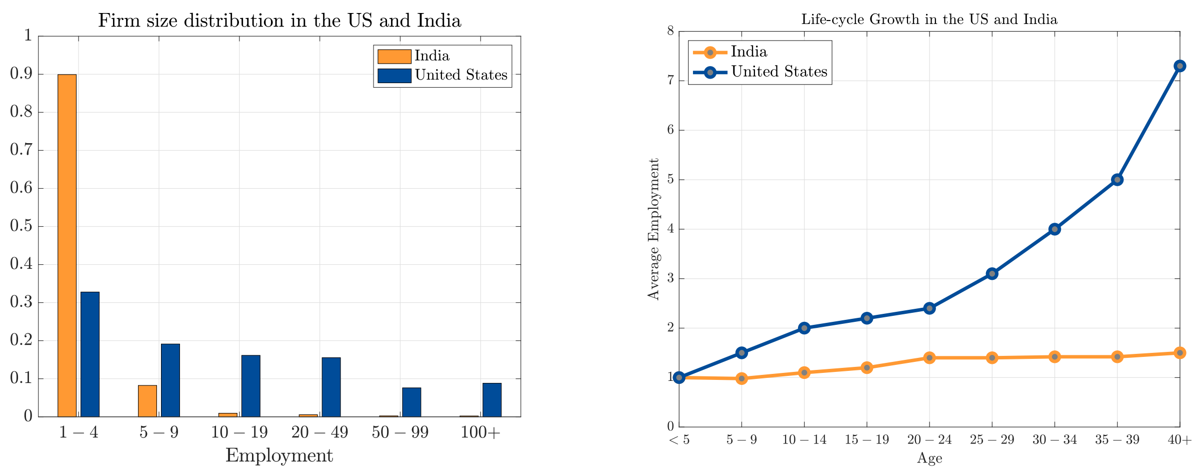
[3] This finding aligns well with that of recent papers showing that the innovation carried out by incumbent firms is an important engine of productivity growth both in the US—see Garcia-Macia et al. (2019)—and in emerging economies like China and Taiwan—see König et al. (2020). Peters (2020) also finds qualitatively similar results for the case of Indonesia.

existing good firms. In contrast, when there is a large pool of high skill entrepreneurs, start-up firms play an important role in the process and barriers to entry are harmful for growth.

Road Map. In the rest of the chapter, we first present some motivating empirical evidence in Section 2. In Section 3, we lay out the model of distance to frontier with firm dynamics and derive the main predictions of the theory. In Sections 4 and 5, we estimate the parameters and perform counterfactual policy experiments. Section 6 concludes.

2 Empirical Evidence

A pervasive feature of economic development is that firms in rich countries look strikingly different from firms in poor countries. They are larger, they experience growth as they age, and a large share of aggregate economic activity is performed in large firms. Figure 1 proposes a comparison between the US and India that illustrates this point well. The left panel shows the firm size distribution. In India, 90% of firms have at most four employees, while only a negligible fraction of Indian firms employ more than 100 workers. In contrast, in the US only one third of firms have less than five employees, while almost 10% of them have more than 100 employees. In short, an organization of economic activity where a sizable share of the labor force works in large corporations is a rich-country phenomenon.



Notes: In the left panel we show the firm size distribution in the US and India. For the US we rely on data from the Business Dynamics Statistics. For India we use the Economic Census. In the right panel we display the employment life-cycle as reported by Hsieh and Klenow (2014).

Figure 1: Firms Dynamics in Developing Countries: India vs the US

The right panel of Figure 1 (based on Hsieh and Klenow (2014)) contrasts the employment life cycle of firms in India and in the US. [4] Again, the firm life cycle is remarkably different in India and in the US. In the US, firms which survive grow as they age. In India, there is very little growth over the life cycle. The evidence in the two panels is intertwined. One of the reasons (and quantitatively a very salient one!) why in the US there are so many large corporations is that firms grow as they age. On the contrary, Indian firms enter small and neither grow nor exit as they age. These patterns are not unique to the case of the US and India. For instance, Bento and Restuccia (2017) document a strong cross-country correlation between income per capita and average size.

The patterns in Figures 1 suggest that there is more creative destruction in rich countries than in poor countries. What are the implications? Is low creative destruction a cause of underdevelopment or is it also in part the effect of it? To cast some light on these questions, we now turn to a novel theory that links the process of firm dynamics to differences in aggregate productivity.

3 Theory

We present a simple model that explains why poor countries have (i) lower creative destruction, (ii) smaller firms, and (iii) a higher share of self-employed subsistence firms. The model also accounts for the fact that at the aggregate level poor countries do not necessarily grow at a slower rate, because they benefit from catch-up growth through the adoption of technologies already in use in other nations. In the steady-state equilibrium of the global economy, all countries grow at the same rate but there are persistent differences in the levels of productivity (i.e., there exists a stationary income distribution) and in the organization of the economy.

Our theory combines elements of Klette and Kortum (2004b) and Acemoglu et al. (2006). From Klette and Kortum (2004b) we borrow the firm-dynamics framework. From Acemoglu et al. (2006) we borrow the idea that countries endogenously specialize in either the adoption of existing knowledge ("imitation") or the creation of new ideas, which result in creative destruction. As a result, our theory endogenously generates cross-countries differences not only in income per capita, but also in the equilibrium distribution of firm size and age-size profile.

Environment. Suppose there are C countries indexed by $c = 1, 2, ..., C$. There is no trade and countries only interact via international technology spillovers described below. The economic structure of each country is similar to Klette and Kortum (2004b). A representative household

[4] Ideally one would want to measure the life-cycle using panel data. Because such data is often hard to find in developing countries, many researchers rely on the above cross-sectional age-size relationship. In a stationary environment these two coincide.

with a unit mass aggregates a continuum one of products in a Cobb Douglas way:

$$\ln Y_t = \int_0^1 \ln y_{it} di.$$

Each product i is produced using labor and is indexed by its productivity q,

$$y_{it} = q_{it} l_{it}.$$

Firms can produce many products and are thus a collection of multiple product lines.

Static Allocation. In equilibrium, each product line i is owned by the monopolist firm that has the highest productivity q_{it}. We assume that each firm can charge a markup μ, which we take as exogenous. This implies that the profits of the monopolist firm producing product i equal

$$\pi_{it} = \frac{\mu - 1}{\mu} p_{it} y_{it} = \frac{\mu - 1}{\mu} Y_t. \tag{1}$$

Total employment in product line i is given by

$$l_i = \frac{y_{it}}{q_{it}} = \frac{1}{\mu} \frac{Y_t}{w_t}.$$

Hence, aggregate output, which we take as the numeraire, is

$$Y_t = \mu L_{Pt} w_t = Q_t L_{Pt}, \tag{2}$$

where $Q_t = \exp\left(\int_{i=0}^1 \ln q_{it} di\right)$. Given these product-line allocations, we can solve for the firm-level allocations. Firm f with n_f products employs

$$l_f = n_f l_i = n_f \frac{1}{\mu} \frac{Y_t}{w_t} = n_f L_{Pt}$$

workers, i.e., employment is proportional to the number of products. Similarly, the total sales of firm f equal $py_f = n_f Y_t$. The number of products n_f thus fully summarizes the state of a firm.

Firm Dynamics, Entry, and Creative Destruction. Productivity growth is directly linked to creative destruction and the process of firm dynamics. A firm that introduces a better technology for product i replaces the incumbent monopolist and adds that product to its portfolio. Conversely, the firm that gets replaced declines in size and possibly exits if i was its sole product line.

Creative destruction stems from both incumbents and entrants. At each point in time a mass z of new firms enter and replace randomly selected incumbent monopolists. Each new firm enters with a single product. Importantly, we assume new firms to be heterogeneous in their type which determines their post-birth innovation potential. A share $1-\delta$ of them are *subsistence firms.* These firms never innovate after entering the market and simply produce their only initial product until they are replaced. By contrast, a share δ of new entrants are high-type firms led by *transformative entrepreneurs* which have the potential to innovate and grow over time.

Each transformative firm can choose the rate X at which it improves the productivity of a randomly selected product by $\gamma_t > 1$ and replaces an incumbent monopolist. Such expansion activities are costly, and we denote these costs (in units of labor) as

$$c(X,n) = \frac{1}{\phi_x} X^{\zeta} n^{1-\zeta} = \frac{1}{\phi_x} x^{\zeta} n, \tag{3}$$

where $\zeta > 1$ and $x = X/n$ denotes the innovation intensity. The parameter ϕ_x governs the innovation efficiency of transformative entrepreneurs and is a key parameter in our analysis.

Because profits and innovation costs only depend on the number of products n, the value function of a transformative entrepreneur, $V_t^T(n)$, solves the following HJB equation:

$$r_t V_t^T(n) - \dot{V}_t^T(n) = n\pi_t - n\tau_t \left[V_t^T(n) - V_t^T(n-1)\right] + \max_x \left\{ nx \left[V_t^T(n+1) - V_t^T(n)\right] - \frac{1}{\phi_x} x^{\zeta} n w_t \right\}, \tag{4}$$

where τ_t denotes the rate of creative destruction at which incumbent monopolists are replaced. The right-hand side of (4) comprises three parts. First, the firm earns the flow profits $n\pi_t$ given in (1). Second, the firm might lose one of its products to other firms, which occurs at the endogenous flow rate $n\tau_t$. Finally, the value function incorporates the option value of expansion: with the flow rate nx, the firm expands into a new market and experiences a capital gain of $V_t(n+1) - V_t(n)$, with the associated costs given by (3).

Along a balanced growth path (BGP), the value function in (4) has a simple solution. In Appendix Section A-1, we establish that $V_t^T(n)$ is given by

$$V_t^T(n) = v_t^T n, \quad \text{where} \quad v_t^T = \frac{\pi_t + (\zeta-1)\frac{1}{\phi_x} x^{\zeta} w_t}{\rho + \tau}, \tag{5}$$

and the optimal innovation rate x is given by

$$x = \left(\frac{v_t^T}{w_t} \frac{\phi_x}{\zeta} \right)^{1/(\zeta-1)}. \tag{6}$$

v_t^T in Equation (5) yields the value of each product for a transformative entrepreneur, which includes the discounted value of current profits and the option value of innovation. The appropriate discount rate comprises the time discounting ρ and the rate of creative destruction τ—the higher τ, the shorter the time-horizon for each product and, hence, the lower its value. Equation (6) shows that v_t^T determines the incentive for transformative firms to innovate: the higher v_t^T relative to the equilibrium wage, the higher the expansion rate of transformative firms.

Subsistence firms own at most one product. The value of an active subsistence firm is akin to the value of a single product for a transformative firm, except that subsistence firms face an arbitrarily large innovation cost, i.e., $\phi_x \to 0$. Hence,

$$v_t^S = \frac{\pi_t}{\rho + \tau}. \tag{7}$$

Equations (5) and (7) highlight an important aspect of our theory: transformative entrepreneurs have a higher private value of owning a product than subsistence firms: $v_t^T > v_t^S$. Because firms only capture a fraction of the social value they create, the gap in the social value of products owned by transformative relative to subsistence firms is even larger. This observation stresses the efficiency-enhancing role of creative destruction originating from transformative firms: in equilibrium, the innovation flow in the economy increases in the share of products owned by transformative firms. Hence, subsistence firms play the role of parasites as argued by La Porta and Shleifer (2009).

Next, consider entry. We assume that entry is subject to a linear technology whereby, if the aggregate entry flow is z, a potential entrant needs to hire $\psi(z)$ workers, where

$$\psi(z) = \frac{1}{\phi_z} z^{\chi}.$$

Here, $\chi \geq 0$ so that there are decreasing returns to scale to entry at the aggregate level. ϕ_z is an inverse measure of the entry cost. Differences in entry cost across countries could be due to either technological or institutional factors, such as red-tape costs, licenses, and other entry barriers. We assume that new entrants don't know their type and always start with a single product. Therefore, free entry requires that the entry cost equal the expected value value of a firm of unknown quality:

$$\frac{1}{\phi_z} z^{\chi} w_t = \delta v_t^T + (1-\delta) v_t^S = \frac{\pi_t + \delta(\zeta - 1)\frac{1}{\phi_x} x^{\zeta} w_t}{\rho + \tau}. \tag{8}$$

Note that the value of entry is akin to the value of a transformative firm, except that the option value of innovation is discounted by δ.

The rate of creative destruction τ is the key endogenous variable. Its equilibrium expression is pinned down by two endogenous flow rates: z (entry) and x (incumbent innovation). Both of them are constant in a BGP. Because transformative firms only innovate in proportion to the products they possess (recall that x is an innovation *intensity*) and the product space is normalized to unity, the rate of creative destruction is given by

$$\tau = z + \sigma^T \times x, \tag{9}$$

where σ^T is the endogenous share of products owned by transformative firms. We derive a closed form expression of σ^T below. Equation (9) highlights that innovative efforts by transformative firms have a multiplier effect. Holding x constant, creative destruction is increasing in the aggregate employment share by transformative firms σ^T. At the same time, we show below that σ^T is increasing in x as transformative firms expand at the expense of subsistence producers.

Aggregate Growth and Distance to Frontier. The product-specific productivity q evolves on a quality ladder. As a new firm innovates and replaces the incumbent producer, quality increases by a factor $\gamma > 1$, i.e., the new productivity q_i' is given by $q_i' = \gamma q_i$. We assume that γ varies across countries and over time depending on the position of each country in the international technology ladder. In particular, for a generic country c,

$$\gamma_{ct} = 1 + \left(Q_{Ft}/Q_{ct}\right)^{\kappa}, \tag{10}$$

where $\kappa > 0$ is a parameter, Q_c is the aggregate productivity of country c, and Q_F is the aggregate productivity of the frontier economy, i.e., the country with the most efficient aggregate technology. For the moment, we simply assume that Q_F grows over time at the constant rate g_F—we later endogenize this. Equation (10) captures a process of knowledge diffusion through international spillovers. The assumption that $\kappa > 0$ incorporates a notion of backwardness and catch-up technology convergence: the further away a country from the technological frontier Q_F, the higher its step size.

Given the rate of creative destruction τ in Equation (9), aggregate productivity growth in country c evolves as follows:

$$g_{ct} = \frac{\dot{Q}_{ct}}{Q_{ct}} = \ln\left(\gamma_{ct}\right) \times \tau_c = \ln\left(1 + \left(\frac{Q_{Ft}}{Q_{ct}}\right)^{\kappa}\right) \times \tau_c. \tag{11}$$

Note that the equilibrium converges to a stationary distribution where all countries grow at the common rate g_F. More formally, the steady state is pinned down by the following system of equations for $c = 1, 2, ..., C$:

$$g_F = \ln\left(1 + \left(\frac{Q_{Ft}}{Q_{ct}}\right)^{\kappa}\right) \times \tau_c, \tag{12}$$

where Q_F/Q_c is the steady-state productivity gap relative to the technology frontier. Note that, when $c = F$, Equation (12) pins down the growth rate of the world economy: $g_F = \ln(2)\tau_F$. It is at this point clear that the frontier economy is the one with the highest rate of creative destruction. The system of equations (12) can alternatively be expressed as:

$$Q_c = \left(exp\left(\frac{\ln(2)\tau_F}{\tau_c}\right) - 1\right)^{-1/\kappa} Q_F. \tag{13}$$

Equation (13) highlights an important implication of our theory: along a balanced growth path, countries are ranked on a global productivity ladder and their rung is fully determined by the rate of creative destruction τ. Countries where creative destruction is large are high up on the ladder and their income per capita is large. Countries with low creative destruction are relatively poor. However, the distribution of income is stationary and all countries grow at the same rate g_F.

The parameter κ determines the speed of convergence. If κ is large, backward countries benefit substantially from their technological backwardness, as the productivity gap translates into a large step size. By contrast, if κ is small, even large differences in productivity do not generate large differences in the step size of creative destruction.

The Organization of the Economy in the BGP. We now turn to the equilibrium determination of τ in each country. In the BGP, the firm size distribution is stationary in each country and countries at grow at a common constant rate. Both wages w_t and the value functions v_t^T and v_t^S grow at the same constant rate. Within each country, the entry flow z and the innovation intensity x are constant. Because the firm size distribution is stationary, the share of transformative products σ^T and, hence, the rate of creative destruction τ is also constant.

To solve for σ^T, let F^S denote the mass of subsistence firms in the economy. At each point in time, $(1 - \delta)\, z$ new subsistence firms are born. Similarly, since all active subsistence firms have just one product, they exit at the rate τ. The stationary mass of subsistence firm is then given by

$$F^S = \frac{(1 - \delta)\, z}{\tau}. \tag{14}$$

Note that faster creative destruction reduces the mass of subsistence firms by inducing faster exit. Since subsistence firms only produces a single product, $\sigma^T = 1 - F^S$. Equation (9) thus implies that the equilibrium rate of creative destruction τ is determined by the condition

$$\tau = z + \left(1 - \frac{(1-\delta)\, z}{\tau}\right) x$$

which solves as

$$\tau = \frac{z+x}{2} + \sqrt{\frac{(z+x)^2}{4} - (1-\delta)\, zx}. \tag{15}$$

Creative destruction is increasing in both z and x: countries with low entry and low innovation from incumbents have little creative destruction in equilibrium. Moreover, holding z and x constant, τ is increasing in the share of transformative firms δ. Finally, we note that (15) implies that $\tau > x$ as long as $\delta < 1$, $z > 0$, and $x > 0$.

Next, we move to the aggregate resource constraint in the labor market. Individuals can either work as production workers, be hired by potential entrants, or be employed in transformative firms to generate further productivity improvements. Given our set of assumptions, the demand for entry labor is given by $L^E = z\psi(z) = z^{1+\chi}\frac{1}{\phi_z}$. Similarly, the demand for researchers from incumbent firms is $L^R = \frac{1}{\phi_x}x^\zeta \sigma^T$. Recalling that $\sigma^T = 1 - F^S$, substituting in the expression of F^S in (14) and normalizing the aggregate labor force to unity yields the following labor market-clearing condition:

$$1 = L^P + z^{1+\chi}\frac{1}{\phi_z} + \frac{1}{\phi_x}x^\zeta \left(1 - \frac{(1-\delta)\, z}{\tau}\right). \tag{16}$$

Equations (15) and (16) still involve the endogenous variables z and x. In Section A-2 in the Appendix, we fully characterize the equilibrium as the solution of a system of three equations in three unknowns for each country: L^P, z and x. In particular, using the free entry condition (8), the optimal rate of incumbent innovation (6) and the labor market clearing conditions, we can solve for z and x as functions of the three key structural parameters of the model: the entry costs $1/\phi_z$, the efficiency of incumbent innovation ϕ_x and the share of high type firms δ. Given the solutions for x and z we can then use (15) to compute the equilibrium rate of creative destruction τ. Proposition 1 summarizes the characterization of the BGP.

Proposition 1. *There exists a unique BGP equilibrium where the entry flow rate z, the innovation rate of transformative firms x, the rate of creative destruction τ, productivity growth g, and the share of production workers L^P are constant and the value functions for subsistence and transformative firms, v_t^S and v_t^T, grow at rate g. In this equilibrium,*

1. *z and x are consistent with free entry* (8) *and the incumbents' optimality condition* (6)*;*

2. *L^P satisfies the labor market clearing condition* (16)*;*

3. τ is consistent with firms' expansion and entry choices (15);

4. the value functions v_t^S and v_t^T are given by (7) *and* (4).

Proof. See Section A-2 in the Appendix. □

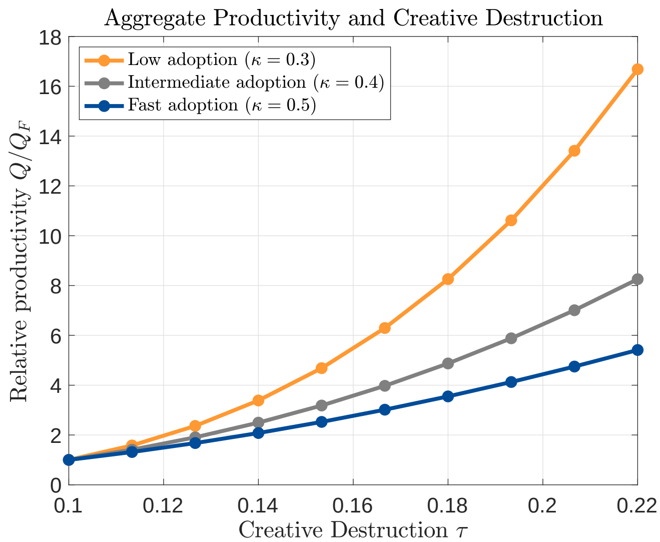
3.1 Taking Stock

Our global economic equilibrium features a stationary productivity distribution across countries owing to international spillovers. Countries with high innovation and creative destruction climb up the global productivity ladder and approach the technological frontier. While progress raises living standards, it also limits the possibilities of further adoption. This is captured by the decline in the step size parameter γ. Thus, as an economy reaches a more advanced stage, its further success hinges on innovation. On average, growth in poor countries is accounted for more by imitation and less by technology adoption: creative destruction events are rare but each of them shifts the productivity frontier by a larger amount. This description of the aggregate dynamics echoes the theory of Acemoglu et al. (2006).

Figure 2 displays the stationary distribution of aggregate productivity Q as a function of the rate of creative destruction. The figure shows three cases corresponding to different values of the parameter κ. The orange line depicts the case where κ is small and adoption is more difficult. In this case, differences in creative destruction lead to large differences in aggregate productivity because adoption is a poor substitute for creative destruction. In this case, increasing creative destruction from 0.1 or 0.2 (which is roughly the difference between the US and India that we estimate below) increases aggregate productivity by a factor of 12. By contrast, if adoption is fast (blue line), the same difference only leads to a productivity difference by a factor of 4. Hence, in line with Acemoglu et al. (2006), adoption allows poor countries to benefit from technological progress and partially catch up even if they cannot generate a lot of innovation and creative destruction themselves. However, if a country wishes to climb up the technological ladder, it must introduce reforms that foster creative destruction.

3.2 Firm Dynamics and Firm Size Distribution

Another important endogenous outcome of the model is the firm size distribution, which is entirely pinned down by the endogenous entry and incumbent innovation flows z and x and by the initial share of transformative firms δ. In this section, we derive closed-form expressions for three moments of the process of firm dynamics that we will use to estimate our theory: the speed of life-cycle



Notes: The figure reports the steady-state relationship between τ and Q as implied by equation (13).

Figure 2: Creative Destruction and Aggregate Productivity

growth shown in the right panel of Figure 1, the share of small firms shown in the left panel of Figure 1 and the equilibrium entry rate.[5]

To start with, recall that, given (z, x, δ), we can compute the rate of creative destruction τ (see (15)), the long-run mass of subsistence firms F^{Sub} (see (14)) and the aggregate share of employment and sales of transformative firms σ^T. In a similar way we can analytically characterize the remaining aspects of the firm size distribution and the process of firm dynamics.

The mass of transformative firms, F^T, is given by $F^T = -\delta \frac{z}{x} \ln\left(1 - \frac{x}{\tau}\right)$. Together with the expression for F^S in (14) this implies that average employment $\bar{\ell}$ and the entry rate er are given, respectively, by

$$\bar{\ell} = \frac{\tau}{z} \frac{1}{1 - \delta\left(1 + \frac{\tau}{x} \ln\left(1 - \frac{x}{\tau}\right)\right)} \quad \text{and} \quad er = \frac{\tau}{1 - \delta\left(1 + \frac{\tau}{x} \ln\left(1 - \frac{x}{\tau}\right)\right)}. \tag{17}$$

Next, we can compute the share of "micro-firms", which we know is pervasive in India (see Figure 1). The share of firms with a single product is given by

$$\vartheta(1) = \frac{1}{1 - \delta\left(1 + \frac{\tau}{x} \ln\left(1 - \frac{x}{\tau}\right)\right)}. \tag{18}$$

Note that $\vartheta(1)$ is decreasing in δ if we hold $\frac{x}{\tau}$ constant and decreasing in $\frac{x}{\tau}$ if we hold δ constant. Thus, the large share of tiny producers in India is driven both by the scarcity of transforma-

[5] We defer to Section A-3 in the Appendix for details on the formal derivation.

tive entrepreneurs and by the fact that incumbent innovation x is only a small share of creative destruction τ (as opposed to entry z). Equations (17) and (18) also imply that

$$er = \tau \times \vartheta(1). \tag{19}$$

To interpret this equation, note that in a stationary equilibrium the entry rate equals the exit rate. Moreover, the share of exiting firms is simply the share of firms with a single product that experience a creative destruction event. With this in mind, (19) can help understand why creative destruction is low in India: exit rates at the firm level are not substantially higher than in the US even though India has a much larger number of small firms close to the exit threshold.

Finally, we can compute the firm's life cycle. Because subsistence firms do not grow and exit at the rate τ, their mass is strictly decreasing with age. In contrast, the life-cycle profile of transformative firms is increasing because these firms can grow. In Section A-3 of the Appendix, we establish that the share of subsistence firms of age a is given by

$$\lambda^{Sub}(a) = \left(1 + \frac{\delta}{1-\delta} \frac{(\tau - x)}{\tau e^{-xa} - x e^{-\tau a}}\right)^{-1}. \tag{20}$$

$\lambda^{Sub}(a)$ is a decreasing function satisfying $\lambda^{Sub}(0) = 1-\delta$ and $\lim_{a\to\infty} \lambda^{Sub}(a) = 0$. This equation captures the dynamic selection process by which transformative entrepreneurs replace subsistence entrepreneurs over time. The share of subsistence firms hinges on $\tau - x$. As creative destruction from incumbent firms declines, selection wanes and the share of subsistence firms remains constant over the life cycle at the unconditional level $1-\delta$. More formally, $\lim_{x\to 0} \lambda^{Sub}(a) = 1-\delta$.

Another facet of the selection process is the size of transformative firms, conditional on survival:

$$E^T[n|a] = 1 + \frac{x}{(\tau - x)}\left(1 - e^{-(\tau - x)a}\right). \tag{21}$$

The average size of transformative firms is increasing with age and such that $lim_{a\to\infty} E^T[n|a] = (1 - x/\tau)^{-1}$. The economic significance of old and large superstar firms hinges on x being large relative to τ—note that, because $x < \tau$, the average size does not grow without bounds.

To link the life-cycle profile of transformative firms to the average life-cycle profile in the economy, we must take into account the selection encapsulated in $\lambda^{Sub}(a)$. The average life-cycle profile is given by

$$E[n|a] = \lambda^{Sub}(a) + \left(1 - \lambda^{Sub}(a)\right) E^T[n|a] = 1 + \left(1 - \lambda^{Sub}(a)\right) \frac{x}{\tau - x}\left(1 - e^{-(\tau - x)a}\right). \tag{22}$$

Equation (22) shows that the slope of the average size profile is the product of the selection term $\left(1 - \lambda^{Sub}(a)\right)$ times the life-cycle slope of transformative entrepreneurs. A shallow profile of life-cycle growth such as the one noted in Figure 1 for Indian firms is a symptom of a weak selection and the slow expansion of transformative firms. In terms of the fundamental parameters, this indicates that India suffers more from the low efficiency of incumbent growth (low ϕ_x) than from large entry barriers (low ϕ_z.)

4 Quantitative Analysis

In this section, we take our model to the data. Given the parsimony of the theory, this analysis is necessarily stylized. We view it as the suggestive first step of a more thorough investigation involving data from multiple countries.

Calibration and Estimation. We calibrate four parameters exogenously. We assume a discount rate ρ of 0.05 and set the growth rate of the technological frontier g_Q^F to 2%. We assume the incumbent innovation cost function to be quadratic. We set $\chi = 0.1$, implying a modest extent of congestion in the entry process.[6]

Then, we estimate the three key parameters ϕ_x, ϕ_z, and δ separately from data for the US and for India. We do so by exactly matching salient moments of the respective firm-size distribution and the process of firm dynamics. In particular, we target the entry rate er, the share of small firms $\vartheta(1)$ and the extent of life-cycle growth $E[n|a]$. Our theoretical analysis establishes that these empirical moments uniquely determine z, x, and τ, from which we can ultimately retrieve the structural parameters δ, ϕ_x, and ϕ_z. Finally, given τ, for both countries, we use Equation (13) to calibrate the parameter κ to match differences in aggregate productivity between the US and India from

$$\frac{Q_{IND}}{Q_{US}} = \left(\frac{exp\left(\frac{g_Q^F}{\tau_{IND}}\right) - 1}{exp\left(\frac{g_Q^F}{\tau_{US}}\right) - 1} \right)^{-1/\kappa}.$$

Note that κ is calibrated residually and that none of the firm-level moments depend on this parameter.

Results. Table 1 reports the data moments, the estimated structural parameters and the associated equilibrium outcomes x, z and τ. We target three firm-level data moments. First, we

[6] Congestion is introduced for computational ease: it ensures that the free entry condition is always binding.

target an 8% entry rate for both countries, following Akcigit et al. (2021). Second, for the share of small firms $\vartheta(1)$, we target a value of 0.9 for the India and a value of 0.4 for the US based on the data in Figure 1. Note that our target for the US slightly exceeds the share of firms with 1–4 employees (which is around 0.32), to adjust for the fact that the smallest size category (which in our theory are firms with a single product) plausibly exceeds 4 employees in the US. Third, we measure life-cycle growth by the size of 10 year old firms relative to new entrants. Because all entrants enter with a single product, this growth rate is given by $E[n|10]$. Figure 1 implies a target moment of 2 for the US, i.e. conditional on survival, firms double in size relative to new entrants. The corresponding moment for India is 1.1. These three data moments are sufficient to exactly identify δ, ϕ_x, and ϕ_z for both the US and India.

Finally, we use differences in aggregate productivity to discipline the adoption parameter κ. As highlighted in Figure 2, for given rates of creative destruction in India and the US, we can infer κ from differences in aggregate productivity. We assume productivity to be eight times as large in the US as in India. This target gap is smaller than the observed differences in income per capita. However, our model abstracts from other sources of income differences like physical capital or human capital. This choice has in any case no consequence on any of the outcomes of interest.

The first panel of Table 1 summarizes the empirical targets. The second panel reports the structural parameters that allow the model to fit those targets. Our model implies that $\delta_{US} > \delta_{IND}$. In the US, roughly 30% of all entering firms are led by transformative entrepreneurs. In India, only 16% of entering firms have growth potential, while 84% of new entrants are stagnant subsistence firms. This finding is qualitatively in line with the results reported in Akcigit et al. (2021). The next two columns report the cost of incumbent expansion (ϕ_x) and the efficiency of entry (ϕ_z). While both are estimated to be more productive in the US, the difference is larger for incumbent innovation: $\phi_x^{US}/\phi_x^{IND} > \phi_z^{US}/\phi_z^{IND}$. In other words, not only has the US a much larger share of transformative firms but also these firms are much more efficient at innovating than the corresponding (transformative) Indian firms.[7]

Finally, in the last three columns, we report the equilibrium outcomes. First and foremost, creative destruction is much higher in the US than in India. Equation (19) shows how we can infer this pattern directly from the data, independently from any model parameters:

$$\frac{\tau_{IND}}{\tau_{US}} = \frac{er_{IND}/\vartheta_{IND}(1)}{er_{US}/\vartheta_{US}(1)} \approx \frac{\vartheta_{US}(1)}{\vartheta_{IND}(1)} = \frac{0.4}{0.9} = \frac{0.09}{0.2} \approx 0.45.$$

Intuitively, entry and exit rates vary little across the two countries, but India has roughly twice

[7] Peters (2020) finds similar patterns between the US and Indonesia.

	Moments				Structural Parameters				Equilibrium Outcomes		
	er	$\vartheta(1)$	$E[n\|10]$	Q	δ	ϕ_x	ϕ_z	κ	z	x	τ
US	0.08	0.4	2	1	0.29	0.280	0.706	0.42	0.002	0.19	0.2
India	0.08	0.9	1.1	1/8	0.16	0.061	0.533	0.42	0.066	0.061	0.09

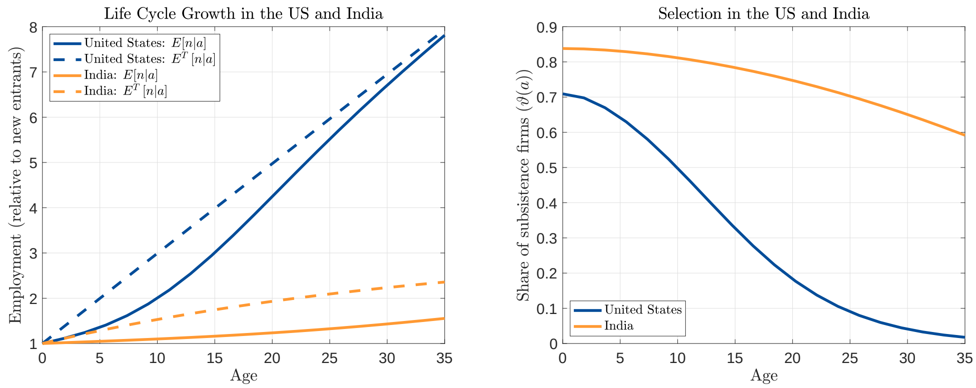
Notes: The table reports the targeted moments (the entry rate er, the share of small firms $\vartheta(1)$, the life-cycle profile of firms at age 10, $E[n|a]$, and the aggregate productivity Q (relative to the US), the structural parameters and the equilibrium outcomes.

Table 1: Calibration: Model Fit and Structural Parameter

as many small firms. Thus, creative destruction turns out to be only half as large. Next, the discrepancies in entry costs and incumbent efficiency translate in both a lower entry flow rate z and a lower innovation flow from incumbent firms x. In the US, innovation from incumbent firms accounts for the vast majority of creative destruction. In India, entry plays a larger role in relative terms. Empirically, we infer these patterns from the entry rate and the life-cycle growth. Because transformative firms expand at rate x and lose products at rate τ, the steep life-cycle profile in the US requires x to be large relative to τ. In contrast, in India creative destruction must be large *relative* to the rate at which incumbent firms expand in order to keep firms small on average. This is achieved by having entrants play a relatively more important role.

In Figure 3 we depict the life-cycle profile of firms in our estimated model. The left panel displays the average size by age $E[n|a]$ (see (22) and the life-cycle profile of transformative firms $E^T[n|a]$ (see (21)) for the US (blue lines) and India (orange lines). Firms in the US grow substantially faster conditional on survival. In India, the average surviving firm barely doubles its size after as long as thirty years. The dashed lines refer to the transformative firms. Naturally, $E^T[n|a] > E[n|a]$. Interestingly, the (few) transformative firms in India do grow. Quantitatively, in the first five years good firms in India do not look very different from the average firm in the US (though they are very different from the transformative US firms). Over time, the average firm looks increasingly similar to a good firm, because subsistence firms are more likely to exit. This shake-out process is shown in the right panel of Figure 3. The share of subsistence firms in the US is already smaller than in India at birth but the difference is at that point modest. Thereafter, the process of selection is much faster in the US. Among thirty year old firms, the share of surviving firms is negligible in the US. Instead, in India as many as 70% of thirty year old firms have no growth potential.

In Figure 4 we display the resulting firm size distribution, captured by the share of firms with n products. The model qualitatively replicates the salient features of the empirical distribution displayed in Figure 1. In India, almost all firms have a single product and the share of firms with more products declines rapidly. In the US there is a also socket of relatively small firms. However,



Notes: In the left panel we depict the average life-cycle profile $E[n|a]$ and the life-cycle profile of high-type firms $E^T[n|a]$. In the right panel we show the share of subsistence firms by age $\vartheta(a)$. The respective models are parametrized according to the parameters reported in Table 1.

Figure 3: Life-cycle Growth in the US and India

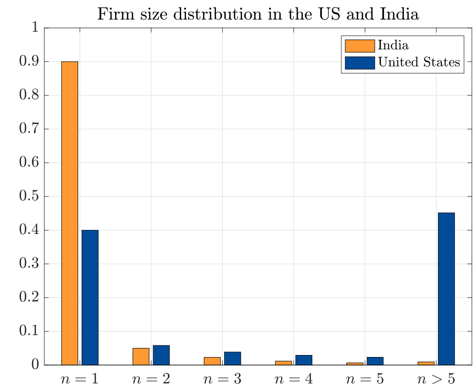
a substantial mass of firms manages to grow large and constitutes the right tail of the firm size distribution.

Cross-country differences in productivity, creative destruction and firm size. Thus far, we have focused on the comparison between India and the US. In this section, we interpret the positive cross-country correlation between aggregate productivity, creative destruction and average firm size through the lens of our model. We view this as a first-step towards a more general analysis that would require data for firm dynamics in multiple countries.

In our theory, creative destruction and aggregate productivity are jointly determined—see Equation (13). All cross-country differences are rooted in differences in entry costs $1/\phi_z$, incumbent innovation efficiency ϕ_x and the share of transformative entrepreneurs δ. To study the role of each of these parameters, we consider the following exercise. Given the structural parameters (ϕ_x, ϕ_z, δ) in the US and India, we generate a synthetic cross-section of countries, whose structural parameters are convex combinations of these parameters. Hence, a country is characterized by a weight parameter ϖ and we then set

$$\phi_x(\varpi) = \varpi\phi_x^{US} + (1-\varpi)\,\phi_x^{IND}$$

and for ϕ_z and δ similarly. Countries with $\varpi \approx 1$ are industrialized counties that are similar to the US. Countries with $\varpi \approx 0$ are similar to India. Given ϖ, we compute the equilibrium allocation and aggregate productivity. By varying ϖ, we can then trace out a synthetic cross-section of countries and study how creative destruction and properties of the firm-size distribution vary with



Notes: The figure shows the equilibrium firm size distribution in India and the US. The respective models are parametrized according to the parameters reported in Table 1.

Figure 4: Firm size distribution in the US and India: Model

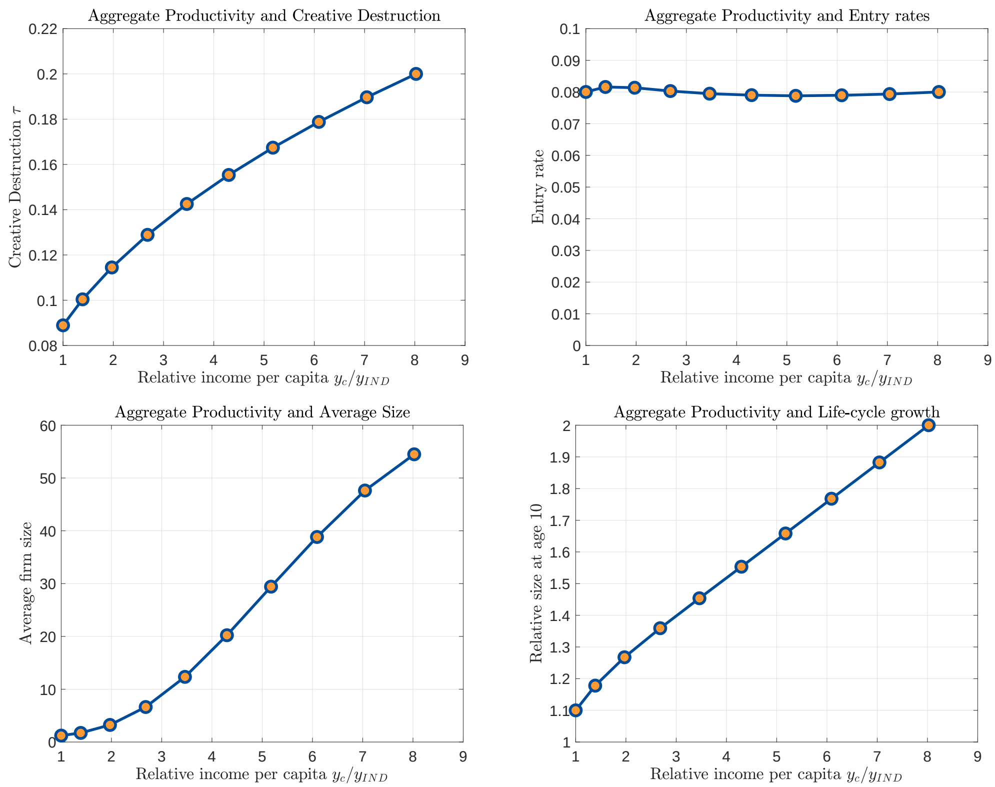
aggregate productivity.

The results of this exercise are presented in Figure 5. We plot creative destruction, the equilibrium entry rate, average firm size and the rate of life-cycle growth against relative income per capita. The top row shows that creative destruction and aggregate entry-and-exit are basically dissociated. Even though richer countries have more creative destruction, entry rates are essentially uncorrelated with income per capita. Equation (19) explains why this is the case. A high rate of creative destruction τ is typically associated with a high degree of incumbent innovation x. Thus, the share of small firms $\vartheta(1)$ tends to be small in the stationary equilibrium.

The lower panels focuses on firm size and the life-cycle profile. The patterns are reminiscent of the empirical findings by Bento and Restuccia (2017) and Hsieh and Klenow (2014), who show that average firm size and life-cycle growth is systematically larger in richer countries. Our theory rationalizes these correlations.

5 Development Policy

We now return to the discussion of development policy. Acemoglu et al. (2006) argue that at an early stage of development government interventions that support selected insider firms can be useful to promote development. The empirical evidence discussed in the introduction suggests that entry barriers are less harmful (and, at instances, beneficial) for growth in poor countries but become a hurdle to technological convergence as countries come closer to the frontier. Our theory abstracts from credit market and contractual frictions that motivate restrictions on competition in Acemoglu et al. (2006). Yet, it allows us to compare the potential gains of different reforms that



Notes: The figure depicts the correlation between aggregate productivity and creative destruction (upper left), the entry rate (upper right), average firm size (lower left) and life-cycle growth, i.e. relative size at age 10 (lower right).

Figure 5: Aggregate Productivity, Creative Destruction and Firm Size

foster creative destruction. Also, we show that a natural extension of the theory can raise a case for targeted industrial policy.

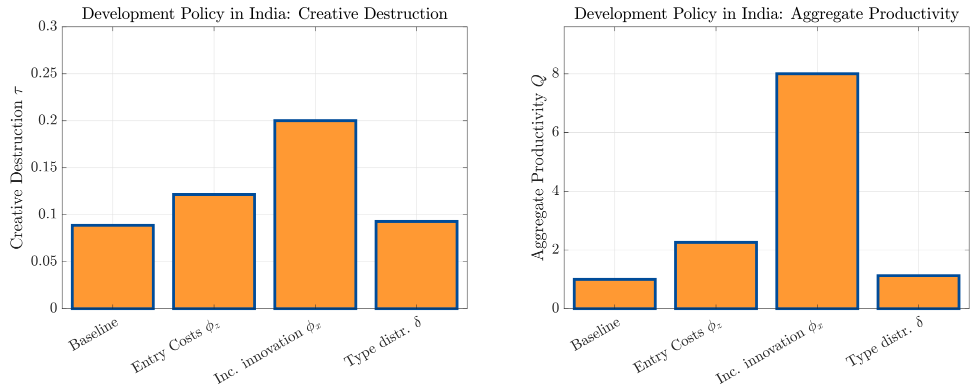
We perform two sets of counterfactual policy exercises. First, we quantify the effect of different policy reforms on productivity and welfare in an economy like India. Next, we study targeted industrial policies which shield transformative entrepreneurs from creative destruction.

Development policy in India. In this section, we consider policy reforms affecting the three fundamental parameters in our theory: entry barriers, innovation efficiency of incumbent firms, and proportion of transformative entrepreneurs. Each margin requires different policy instruments. Entry barriers pertain to the area of industrial policy. The second and third margin are related to human capital formation. In addition, the expansion of incumbent firms has implications for innovation policy (e.g., subsidies to R&D) and for policies regulating the growth of incumbent firms—India has a history of pervasive legal restrictions on the expansion of existing firms in order to protect smaller firms.

Before performing the counterfactual experiments, it is useful to relate our analysis to recent empirical studies of economic reforms in India based on a difference-in-difference methodology. Martin et al. (2017) study the effect of policy reforms reducing small-scale industry promotion in India throughout the early 2000s. Prior to reform, the policy reserved specific production lines for small and medium enterprises. The authors exploit variation in the timing of dereservation and find that districts more exposed to dereservation experienced higher employment and output growth. Aghion et al. (2005 and 2008) study an earlier set of reforms dating back to the late 1980s that dismantled the License Raj—a policy comprising both entry barriers and constraints on the expansion of existing firms. The studies document significant differential effects across Indian states which depend on local labor market regulations. The aggregate effects on employment and growth are instead more ambiguous.

We take the estimated model for India and then individually set the entry cost, the efficiency of incumbent growth, and the share of transformative entrepreneurs to the respective values in the US. We then calculate the equilibrium rate of creative destruction and aggregate productivity for each of the three scenarios.

Figure 6 summarizes the results. The left panel displays creative destruction in the India baseline estimated model and in each of the counterfactuals. Creative destruction is higher in all three scenarios, suggesting that entry costs, the low efficiency of incumbent firms, and the small share of transformative entrepreneurs play all a role in explaining low creative destruction in India. Quantitatively, it is incumbent innovation efficiency ϕ_x that plays the most important role. While reducing entry costs or increasing the share of high type firms increases creative



Notes: The figure depicts the counterfactual effects of (i) lowering entry costs to the level in the US, (ii) raising incumbent innovation efficiency to the level of the US and (iii) increasing the share of transformative types to the US level. All other parameters are held constant. In the left panel we report the respective change in creative destruction τ. In the right panel we report the change in aggregate productivity Q.

Figure 6: Development policy in India

destruction by 10% to 50%, creative destruction would almost double if Indian firms had the same capability to expand through innovation as their US counterparts. The right panel plots the effect on aggregate productivity. We see once again the dominant role of incumbent innovation. While India's productivity would almost double if entry costs were as low as in the US, productivity would increase by a factor of eight if Indian firms had the same innovation capability as US firms. Note that the effects of increasing the share of high types in India by itself is modest, reflecting the fact that transformative firms in India are substantially less productive than in the US.

Targeted Industrial Policy. In the counterfactual experiments of Figure 6, barriers to entry are always harmful for growth. However, one might argue that industrial policy has more refined tools to restrict competition than imposing entry barriers across the board. A benevolent government might have access to some information that allows it to selectively support some firms at the expense of others. Our theory yields a potential rationale of such policies: because transformative firms actually use the products they own as an input to further innovation, the government might want to protect them. And because transformative firms are on average larger and older than subsistence firms, such policy has a flavor of the government protecting old and large producers.

To gauge the potential welfare impact of such policies, we extend our model to allow for *governmental protection*. To capture this idea we augment our model with the assumption that the government can block some successful product innovations that would lead to the replacement of transformative entrepreneurs. More specifically, suppose that the equilibrium degree of creative destruction is τ and that the government can shield transformative products from a share ϱ of such innovations. Hence, products owned by transformative firms face a business stealing threat of $(1-\varrho)\tau$, while subsistence producers face a replacement rate of τ.

Such a policy has costs and benefits. On the one hand, it protects better firms from being replaced, thereby increasing the market share of transformative firms. In addition, the reduction in the risk of being replaced encourages transformative firms to invest in innovation and expand. Both of these mechanisms tend to increase aggregate productivity and welfare. On the other hand, such policies have also detrimental effects. First of all, the government intervention mechanically reduces productivity growth by disallowing some product innovations. If F^S denotes the share of products produced by subsistence firms and τ is the rate of attempted creative destruction, the rate of *realized* creative destruction $\tau^\star$ is given by

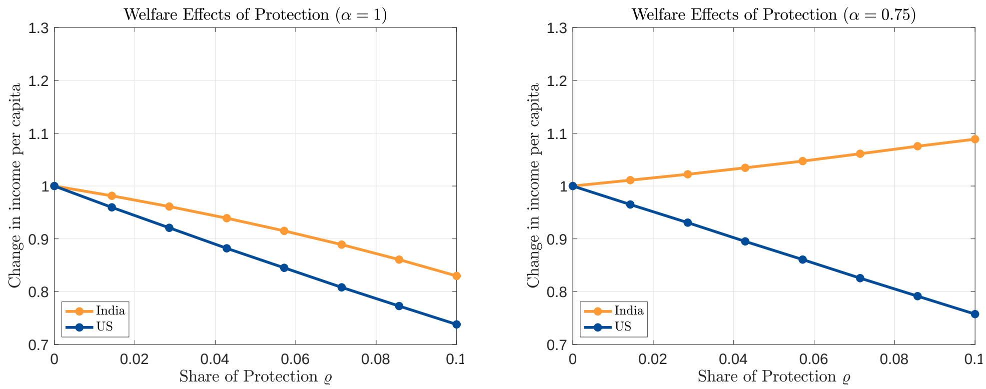
$$\tau^\star = \left(F^S + (1-F^S)(1-\varrho)\right)\tau. \tag{23}$$

Second, such forms of market protection reduce the expected return of innovation—both for transformative firms and potential entrants—because firms anticipate that a share $(1-F^S)\varrho$ of their innovations will be blocked.

In Section A-3.1 in the Appendix we show how to extend our theory with this form of industrial policy. Even though this changes the characterization of the value function and the firm size distribution, we can still compute the equilibrium allocations analytically. We can then easily perform policy counterfactuals and study the welfare impact for different choices of the share of protection ϱ.

The left panel of Figure 7 shows the welfare change as a function of ϱ for both the US and India. We always express welfare relative to the laissez-faire equilibrium, i.e. $\varrho = 0$. The figure highlights two results. First, protection policies reduce welfare in both countries. Even though the share of transformative firms increases in both economies, the adverse effects of blocking creative destruction dominate these positive selection effects. Second, the consequences of such policies are significantly worse in the US than in India. The reason is that the US economy already generates a large degree of selection without any governmental protection for transformative firms. Hence, in an advanced economy, industrial policy shields transformative firms from other transformative firms, thereby reducing productivity growth without any significant gain from selection. The situation is different in India, where most firms are subsistence firms and the bulk of overall creative destruction stems from entrants.

The targeted industrial policy just discussed has different lower net costs in India than in the US but is harmful in both countries. A further extension that can reconcile our theory with the hypothesis of Acemoglu et al. (2006) is that innovation stemming from transformative vs.



Notes: The figure depicts the change in welfare as a function of the degree of protection ϱ. We normalize welfare in the baseline ($\varrho = 0$) to unity. In the left panel we assume the innovation step size to be the same for transformative and subsistence firms. In the right panel we assume that subsistence firms have a 25% smaller step size than transformative firms.

Figure 7: The Welfare Impact of Protection Policies

subsistence firms contributes differentially to productivity growth. In particular, we assume that the productivity step size of transformative innovations is γ while that of subsistence firms is only $\alpha\gamma$, where $\alpha \leq 1$. This model nests our benchmark model as a particular case where $\alpha = 1$. As α becomes smaller we approach a limit case in which subsistence firms can simply copy the technology of transformative firms without contributing at all to productivity growth. We refer again to Section A-3.1 in the Appendix for the details of the formal analysis of this case.

The right panel of Figure 7 shows the results of this case. We assume $\alpha = 0.75$, i.e. innovations by subsistence firms contribute 25% less to productivity growth than the one by transformative firms. The policy has now opposite-sign effects at different stages of development. While in the US the industrial policy still reduces creative destruction, aggregate productivity and welfare, in India it increases them.

The source of the stark discrepancy is the following. In the US, only a small share of products are produced by subsistence firms in equilibrium and most of aggregate growth stems from existing firms, which are transformative. The innovation efficiency of subsistence firms has negligible effects on productivity and welfare. Note that the two panels have the same y-axis, indicating that the welfare impact in the US is insensitive to α.

In India, the share of subsistence firms is large and accounts for a large proportion of the entrants. If the innovations of such firms upon entry is low, their role as "parasites" dominates and policies protecting the few transformative firms can be beneficial.

6 Conclusions

The positive relationship between aggregate productivity and firm size is a salient feature of economic development. In this chapter, we argue that this pattern is the natural implication of a simple model of creative destruction where cross-country income differences are determined by the rate of creative destruction of each economy. Countries where existing firms can grow easily have more creative destruction, are richer, have bigger firms and weed out less efficient producers quickly. Countries where even the best firms grow slowly have little creative destruction, are relatively poor and feature many small firms which manage to survive for a long time.

The theory also bears positive and normative implications about the importance of creative destruction at different stages of development. In line with Acemoglu et al. (2006), our theory predicts that growth in poorer economies hinges more on imitation and technology adoption and less on innovation, churning, and creative destruction. The latter account for a larger share of productivity growth in more advanced economies.

At the normative level, a calibrated version of the theory suggests that barriers to entry may not be the main culprit for low productivity in developing countries. More important is the shortage of dynamic firms that may reflect the human capital of entrepreneurs and managers, as well as the existence of social norms or explicit legal restrictions that limit the growth of the most productive firms. As an example of the latter, Akcigit et al. (2021) highlight the importance of cross-country differences in the efficiency of managerial delegation.

We study the role of a stylized industrial policy that selectively protects some firms in the economy. While in the benchmark estimated model this type of industrial policy harms creative destruction and growth, the result can be reversed in poor economies where most entrant firms replace incumbents while generating only small productivity gains and have no expansion potential after entry. In this case, the business stealing from low-productivity firms can be a major deterrent of innovation from higher-quality incumbent firms. Then, a benevolent government could wish to shield the most dynamic firms from competition. This situation is typical of economies where self-employment and entrepreneurship is a last resort for people on the margin of formal economic activity rather then the choice of the most talented and innovative individuals in society aspiring to success.

The theory presented in this chapter is very stylized. Likewise, the quantification is based on the comparison between only two countries, India and the US. Future research can build on the insights of this work and carry out more formal estimation using data from many countries. A potential criticism of the theory is that it assigns no role to neck-and-neck competition and escape innovation. Incumbent firms cannot reduce the risk of replacement by improving the product in their current product line. Some of the predictions can change if this realistic feature is incorporated, although we expect the main insights to be robust. The theory also abstracts from distributional effects: creative destruction generates winners and losers, and taking this into consideration is important for normative analysis. We also abstract from frictions and market incompleteness. We hope that these and other limitations can be addressed by future research.

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APPENDIX

A-1 Optimal incumbent innovation and the value function

The value function of a high type firm is given by

$$rV(n) - \dot{V}(n) = \pi n + \max_x \left\{ xn \left(V(n+1) - V(n) \right) - \phi n x^{\zeta} w_t \right\} + \tau n \left(V(n-1) - V(n) \right).$$

Conjecture that

$$V_t(n) = v_t n.$$

Then

$$r v_t n - \dot{v}_t n = \pi n + n \max_x \left\{ x v_t - \frac{1}{\phi_x} x^{\zeta} w_t \right\} - \tau n v_t.$$

Hence, homogenous of degree n. Hence, v_t solves

$$r v_t - \dot{v}_t = \pi + \max_x \left\{ x v_t - \frac{1}{\phi_x} x^{\zeta} w_t \right\} - \tau v_t.$$

Optimal x solves

$$v_t - \frac{1}{\phi_x} \zeta x^{\zeta - 1} w_t = 0$$

so that

$$x = \left(\frac{v_t}{w_t} \frac{\phi_x}{\zeta} \right)^{\frac{1}{\zeta - 1}}$$

Along BGP: $v_t \propto w_t$ so that x is constant. Also

$$\max_x \left\{ x v_t - \frac{1}{\phi_x} x^{\zeta} w_t \right\} = (\zeta - 1) \frac{1}{\phi_x} x^{\zeta} w_t$$

v_t grows at rate g_y. Euler equation requires $r - g = \rho$. Hence,

$$r v_t - \dot{v}_t = \pi + (\zeta - 1) \frac{1}{\phi_x} x^{\zeta} w_t - \tau v_t.$$

so that

$$v_t^T = \frac{\pi_t + (\zeta - 1) \frac{1}{\phi_x} x^{\zeta} w_t}{\rho + \tau}$$

Similarly, the value of a low type

$$v_t^{Sub} = \frac{\pi_t}{\rho + \tau}.$$

A-2 Steady state equilibrium

Using the expression for profits and aggregate output (see (1) and (2)), firms' profits relative to the equilibrium wage are given by

$$\frac{\pi_t}{w_t} = \frac{\mu - 1}{\mu} \frac{Y_t}{w_t} = (\mu - 1) L_t^P.$$

The free entry condition in (8) thus implies that

$$\frac{1}{\phi_z} z^{\chi} = \frac{\frac{\pi_t}{w_t} + \delta (\zeta - 1) \frac{1}{\phi_x} x^{\zeta}}{\rho + \tau} = \frac{(\mu - 1) L_t^P + \delta (\zeta - 1) \frac{1}{\phi_x} x^{\zeta}}{\rho + \tau}. \tag{A-1}$$

Similarly, optimal incumbent innovation is given by (see (6))

$$x = \left(\frac{v_t^T}{w_t} \frac{\phi_x}{\zeta} \right)^{1/(\zeta - 1)} = \left(\frac{(\mu - 1) L_t^P + (\zeta - 1) \frac{1}{\phi_x} x^{\zeta}}{\rho + \tau} \frac{\phi_x}{\zeta} \right)^{1/(\zeta - 1)}. \tag{A-2}$$

Labor market clearing implies that

$$L = L^P + L^E + L^R = L^P + z^{1+\chi} \frac{1}{\phi_z} + \frac{1}{\phi_x} x^{\zeta} \left(1 - \frac{(1 - \delta) z}{\tau} \right). \tag{A-3}$$

Together with the definition of the rate of creative destruction in (15), equations (A-1), (A-2) and (A-3) are three equations in three unknowns $\left(x, z, L^P\right)$. The solution to these equations fully determines the equilibrium.

A-3 The process of firm dynamics and the firm size distribution

Let δ,z and x be given. Given (δ, x, z) we can solve for (i) the number of firms and the entry rate, (ii) the rate of life-cycle growth and (iii) the share of small firms.

The Number of Firms and the Entry Rate

The mass of subsistence firms is given in (14). To solve for the mass of transformative firms we have to solve for for the firm size distribution. Let $F_t^T(n)$ denote the mass of transformative firms with n products at time t. For $n > 1$, this mass solves the differential equation

$$\frac{\partial F_t^T(n)}{\partial t} = \tau (n + 1) F_t^T(n + 1) + x (n - 1) F_t^T(n - 1) - (\tau + x) n F_t^T(n).$$

Similarly, for $n = 1$, it solves the differential equation

$$\frac{\partial F_t^T(1)}{\partial t} = \tau 2 F_t^T(2) + \delta z - (\tau + x) F_t^T(1).$$

In a stationary equilibrium, it is easy to show that that the mass of transformative firms is given by

$$F_t^T = -\frac{\delta z}{x} \ln\left(1 - \frac{x}{\tau}\right),$$

and the share of transformative firms with n products is

$$\omega_t(n) = \frac{1}{F_t^T} \frac{\delta z}{x} \frac{1}{n} \left(\frac{x}{\tau}\right)^n.$$

The entry rate is therefore given by

$$\text{Entry-Rate} = \frac{z}{F_t^T + F_t^S} = \frac{z}{\frac{(1-\delta)z}{\tau} - \frac{\delta z}{x} \ln\left(1 - \frac{x}{\tau}\right)}. \tag{A-4}$$

The rate of life-cycle growth

To calculate the life-cycle growth in our economy, let $\lambda^S(a)$ denote the share of subsistence firms for a cohort of age a. The expected number of products as a function of age is thus given by

$$E[n|a] = \lambda^S(a) E^S[n|a] + \left(1 - \lambda^S(a)\right) E^T[n|a] = \lambda^S(a) + \left(1 - \lambda^S(a)\right) E^T[n|a], \tag{A-5}$$

where $E^S[n|a]$ and $E^T[n|a]$ denote the average number of products as a function of age for subsistence and transformative firms. Of course, $E^S[n|a] = 1$. We now compute $E^T[n|a]$ and $\lambda^S(a)$.

Consider first $E^T[n|a]$. For a transformative firm, the setup is the same as in the baseline model of Klette and Kortum (2004b). Hence,

$$E^T[n|a] = \frac{1}{1 - \gamma(a)} \quad \text{where} \quad \gamma(a) = \frac{x\left(1 - e^{-(\tau - x)a}\right)}{\tau - x e^{-(\tau - x)a}}. \tag{A-6}$$

Now consider $\lambda^S(a)$. Because subsistence firms exit at rate τ, the probability of a subsistence firm to survive until age a is given by

$$p^S(a) = e^{-\tau a}.$$

For transformative firms we can again exploit the results from Klette and Kortum (2004b). The probability for a transformative firm to have 0 products at age a is given by

$$p_0^T(a) = \frac{\tau}{x} \gamma(a),$$

where $\gamma(a)$ is given in (A-6). Hence, the share of transformative firms that are still alive at age a

is given by

$$p^{T}(a) \equiv 1-p_{0}^{T}(a)=1-\frac{\tau}{x}\gamma(a)=\frac{(\tau-x)\, e^{-(\tau-x)a}}{\tau-xe^{-(\tau-x)a}}$$

Given that the initial mass of subsistence (transformative) firms is given by $(1-\delta)\, z$ and δz we have

$$\lambda^{S}(a) = \frac{(1-\delta)\, zp^{S}(a)}{(1-\delta)\, zp^{S}(a)+\delta zp^{T}(a)}=\frac{1}{1+\frac{\delta}{1-\delta}\frac{(\tau-x)e^{xa}}{\tau-xe^{-(\tau-x)a}}}. \quad \text{(A-7)}$$

The share of small firms

The mass of firms with a single product is given by

$$F_{t}(1)=F^{Sub}+F_{t}^{T}\omega_{t}(1)=\frac{(1-\delta)\, z}{\tau}+\frac{\delta z}{x}\frac{x}{\tau}=\frac{z}{\tau}$$

Hence, the share of firms with a single product

$$\varrho(1)=\frac{z/\tau}{\frac{(1-\delta)z}{\tau}-\frac{\delta z}{x}\ln\left(1-\frac{x}{\tau}\right)}=\frac{1}{1-\delta-\delta\frac{\tau}{x}\ln\left(1-\frac{x}{\tau}\right)}=\frac{1}{1-\delta\left(1+\frac{\tau}{x}\ln\left(1-\frac{x}{\tau}\right)\right)}. \quad \text{(A-8)}$$

Note that a higher δ reduces $\varrho(1)$ holding τ constant. In addition, for given x and τ, $\varrho(1)$ does not depend on z.

The firm size distribution

The mass of firms with n products, $F(n)$, is given by

$$F(n)=\begin{cases}\frac{z}{\tau} & \text{if} n=1\\ \frac{\delta z}{x}\frac{1}{n}\left(\frac{x}{\tau}\right)^{n} & \text{if } n\geq 2\end{cases}.$$

Hence, total employment by firms with n products is given by

$$E(n)=nF(n)\, l=\begin{cases}\frac{z}{\tau}l & \text{if} n=1\\ \frac{\delta z}{x}\left(\frac{x}{\tau}\right)^{n}l & \text{if } n\geq 2\end{cases},$$

where l denotes aggregate employment per product. Hence, the share of employment in firms with n products is given by

$$q(n)=\frac{E(n)}{\sum_{n=1}^{\infty}E(n)}.$$

Note that

$$\sum_{n=1}^{\infty}E(n)=\left(\frac{1-\delta}{\tau}+\frac{\delta}{\tau-x}\right)zl.$$

Hence,

$$q(n) = \begin{cases} \frac{1}{\tau}\frac{1}{\left(\frac{1-\delta}{\tau}+\frac{\delta}{\tau-x}\right)} & \text{if } n=1 \\ \frac{\frac{\delta}{x}\left(\frac{x}{\tau}\right)^n}{\left(\frac{1-\delta}{\tau}+\frac{\delta}{\tau-x}\right)} & \text{if } n\geq 2 \end{cases}.$$

A-3.1 The Model with Industrial Policy

Suppose the government can enact a simple industrial policy of the following form: If a creative destruction event happens on a product produced by a transformative firm, it can "block" the innovation. While this shields high-type firms, it also has a cost in that the productivity frontier does not evolve. To incorporate this idea in our model, suppose that a fraction ϱ of all creative destruction for high type products is blocked. As before we denote by z and x the flow rates of attempted entry and incumbent expansion and by F^S the share of subsistence products. The aggregate amount of attempted creative destruction is thus still given by

$$\tau = z + \left(1 - F^S\right)x.$$

However, while subsistence firms are indeed replaced at rate τ, high type firms are replaced are replaced at rate $(1-\varrho)\tau$. Then total flow of successful creative destruction is therefore given by

$$\tau^* = \tau F^S + \tau(1-\varrho)\left(1-F^S\right) = \tau\left(1-\varrho+\varrho F^S\right).$$

Given this new structure, the mass of subsistence firms F^S solves the steady-equation

$$\tau F_t^S = (1-\delta)z\left(1-\varrho+\varrho F^S\right).$$

Hence,

$$F^S = \frac{(1-\delta)z(1-\varrho)}{\tau-(1-\delta)z\varrho}.$$

Note that if $\varrho = 0$ we have $F^S = \frac{(1-\delta)z}{\tau}$ as before. Note also that if $\varrho = 1$ we have $F^S = 0$. Intuitively: if low type firms steal only from low type firm but high type firms also steal from low type firms, in the long-run there will be only high type firms and creative destruction will be zero as $\tau^* = \tau F^S = 0$.

The rate of attempted creative destruction is thus given by

$$\tau = z + \left(1 - \frac{(1-\delta)z(1-\varrho)}{\tau-(1-\delta)z\varrho}\right)x.$$

This determines τ as a function of (z, x, δ) and the parameter ϱ. In particular, the solution for τ is

$$\tau = \frac{((1-\delta)\varrho+1)z+x}{2} + \sqrt{\frac{(((1-\delta)\varrho+1)z+x)^2}{4} - (x+\varrho z)(1-\delta)z}.$$

The rate of actual creative destruction that generates productivity increases τ^* is - given the solution for F^S - given by

$$\tau^* = \frac{(1-\varrho)\,\tau^2}{\tau - (1-\delta)\,z\varrho}$$

Hence, given τ we can directly compute τ^*.

The firm size distribution To compute the firm size distribution, focus on transformative firms. Such firms lose products at rate $\tau^T = (1-\kappa)\,\tau$. Similarly, they expand at rate

$$x^T = x\left(1-\varrho+\varrho F^S\right),$$

as a fraction ϱ of all innovations that threaten to replace high-type firms are blocked. Similarly, the entry flow of transformative firms is given by

$$z^T = \delta z\left(1-\varrho+\varrho F^S\right).$$

Given $\left(\tau^T, x^T, z^T\right)$, all the theoretical results by KK apply. Hence, the number of transformative firms is given by

$$F^T = -\frac{z^T}{x^T}\ln\left(1-\frac{x^T}{\tau^T}\right)$$

and and the share of transformative firms with n products is

$$\omega(n) = \frac{1}{F^T}\frac{z^T}{x^T}\frac{1}{n}\left(\frac{x^T}{\tau^T}\right)^n.$$

Aggregate productivity growth To determine the rate of aggregate productivity growth, we generalize our baseline analysis in the following way (as outlined in the main text): if transformative firms increase the productivity frontier with a step size of γ^T, subsistence firms only generate a step size of $\alpha\gamma^T$, where $0 \leq \alpha \leq 1$. This implies that productivity growth is given by

$$\frac{\dot{Q}_t}{Q_t} = \ln\left(\gamma^T\right)\tau^* + \ln(\alpha)\,(1-\delta)\,z\left(1-\varrho+\varrho F^S\right).$$

Note that $\alpha < 1$ so that $\ln(\alpha) < 0$. Hence, given τ^*, productivity growth is decreasing in the extent of low-type innovation $(1-\delta)\,z$.

The level of productivity is therefore given by

$$Q = \left(\exp\left(\frac{g_Q - \ln(\alpha)\,(1-\delta)\,z\left(1-\varrho+\varrho F^S\right)}{\tau^*}\right) - 1\right)^{-1/\kappa} Q_F.$$

Given τ^* and F^S we can solve for Q as a function of parameters.

The value function and optimal entry and innovation effort The value function of a high type firm is given by

$$\begin{aligned} rV(n) - \dot{V}(n) &= \pi n + \max_x \left\{ x\left(1 - \varrho + \varrho F^S\right) n \left(V(n+1) - V(n)\right) - \phi n x^{\zeta} w_t \right\} \\ &\quad + (1-\varrho)\,\tau n \left(V(n-1) - V(n)\right). \end{aligned}$$

Conjecture again that $V_t(n) = v_t n$. Then

$$r v_t n - \dot{v}_t n = \pi n + n \max_x \left\{ x\left(1 - \varrho + \varrho F^S\right) v_t - \frac{1}{\phi_x} x^{\zeta} w_t \right\} - (1-\varrho)\,\tau n v_t.$$

This implies that v_t solves

$$r v_t - \dot{v}_t = \pi + \max_x \left\{ x\left(1 - \varrho + \varrho F^S\right) v_t - \frac{1}{\phi_x} x^{\zeta} w_t \right\} - (1-\varrho)\,\tau v_t.$$

The optimal rate of incumbent innovation x is given by

$$x = \left(\frac{v_t\left(1 - \varrho + \varrho F^S\right)}{w_t} \frac{\phi_x}{\zeta} \right)^{\frac{1}{\zeta - 1}}. \tag{A-9}$$

Because v_t grows at rate $g_y = r - \rho$ along a BGP, the value function for transformative firms is given by

$$v_t^T = \frac{\pi_t + (\zeta - 1)\frac{1}{\phi_x} x^{\zeta} w_t}{\rho + \tau(1-\varrho)}.$$

Note that the policy parameter ϱ appears in two places. First a higher ϱ reduces the discount rate of transformative firms through a lower rate of replacement. Second, ϱ also affects the optimal rate of innovation x and hence the value of transformative firms.

Because the products of subsistence firms are not affected, the value of subsistence firms is given by

$$v_t^S = \frac{\pi_t}{\rho + \tau}$$

as before. Given v_t^S and v_t^T, the free entry condition is given by

$$\frac{1}{\phi_z} z^{\chi} = \left(1 - \varrho + \varrho F^S\right) \frac{E[V]}{w_t} = \left(1 - \varrho + \varrho F^S\right) \left(\delta \frac{v_t^T}{w_t} + (1-\delta) \frac{v_t^S}{w_t} \right). \tag{A-10}$$

Equilibrium Note that profits are still given by

$$\pi_t = (\mu - 1)\, w_t L_t^P,$$

so that the value functions (relative to the wage) are given by

$$\frac{v_t^T}{w_t} = \frac{(\mu - 1) L_t^P + (\zeta - 1) \frac{1}{\phi_x} x^\zeta}{\rho + \tau (1 - \varrho)} \quad \text{and} \quad \frac{v_t^S}{w_t} = \frac{(\mu - 1) L_t^P}{\rho + \tau}.$$

Labor market clearing still requires that

$$\begin{aligned} L_t^P &= 1 - z^{1+\chi} \frac{1}{\phi_z} - \frac{1}{\phi_x} x^\zeta \left(1 - F^S\right) \\ &= 1 - z^{1+\chi} \frac{1}{\phi_z} - \frac{1}{\phi_x} x^\zeta \left(1 - \frac{(1-\delta) z (1-\varrho)}{\tau - (1-\delta) z \varrho}\right). \end{aligned} \tag{A-11}$$

Hence, as before we can use the labor market clearing condition (A-11), the free entry condition (A-10) and the optimality condition for incumbent expansion (A-9) to compute the equilibrium as a function of parameters.