

# The Gains from Input Trade in Firm-Based Models of Importing by Joaquin Blaum, Claire Lelarge and Michael Peters

## Online Appendix (Not for Publication)

This Appendix contains the following additional results and material:

1. Details about the identification of the Input-Output Matrix  $\Xi$ ,
2. Estimates of the production function coefficients,
3. A description of the bootstrap procedure,
4. An extension of the welfare equation (40) to a multi-sector environment,
5. Details about the algorithm used to calibrate the model of Section 4.

### A.1 Identification of the Input-Output and Demand Structure

We use the French input-output tables from the OECD to discipline the demand parameters  $[\alpha_s]$  and the matrix of input-output linkages  $\Xi$ . To determine  $\Xi$ , we focus on the intermediate supply from each industry  $j$  to each industry  $s$ . We abstract from any taxes and subsidies. As  $\Xi$  can be identified from expenditure shares by sourcing sector, see (25), we set

$$\zeta_j^s = \frac{\text{Intermediate supply from industry } j \text{ to industry } s}{\text{Intermediate consumption at final prices from industry } s}.$$

That is,  $\zeta_j^s$  measures the importance of sector  $j$  in the production process of sector  $s$ . By construction, this ensures that  $\sum_{j=1}^S \zeta_j^s = 1$  for all  $s$ . We arrange the input-output matrix so that the columns contain the distribution of expenditure for the different sectors:

$$\Xi = \begin{bmatrix} \zeta_1^1 & \zeta_1^2 & \zeta_1^S \\ \zeta_2^1 & & \\ \zeta_S^1 & & \zeta_S^S \end{bmatrix}.$$

To determine  $[\alpha_s]$ , we also use the input-output tables as they contain information on the composition of final demand. Since there is no trade in final goods in the theory, we exclude any exports and imports in final goods in the data. More specifically, the input-output tables report final consumption expenditure by households on sector  $j$ , denoted by  $HHFC_j$ . Following (25), we hence set

$$\alpha_s = \frac{HHFC_s}{\sum_{j=1}^S HHFC_j}.$$

Direct Data						Aggregation		
ISIC	$\alpha$	Value Added	Intermediate Purchases	$\gamma$	$\Lambda$	Coarse Classification	$\alpha$	$\gamma$
1	$\alpha_1$	$VA_1$	$X_1$	$\frac{X_1}{X_1+VA_1}$	0	Non-Manufacturing	$\alpha_S$ from (81)	$\gamma_S$ from (82)
2	$\alpha_2$	$VA_2$	$X_2$	$\frac{X_2}{X_2+VA_2}$	0			
3					0			
...					0			
10	$\alpha_{10}$	$VA_{10}$	$X_{10}$	Estimate from micro-data	"Read off" from micro-data	Manufacturing	$\alpha_{10}$	$\gamma_{10}$
16								
...								
37	$\alpha_{37}$	$VA_{37}$	$X_{37}$				$\alpha_{37}$	$\gamma_{37}$
40	$\alpha_{40}$	$VA_{40}$	$X_{40}$	$\frac{X_{40}}{X_{40}+VA_{40}}$	0	Non-Manufacturing	$\alpha_S$ from (81)	$\gamma_S$ from (82)
...					0			
99	$\alpha_{99}$	$VA_{99}$	$X_{99}$	$\frac{X_{99}}{X_{99}+VA_{99}}$	0			

Table 14: Measurement of  $\alpha_s$  and  $\gamma_s$

The OECD input-output tables report their data at the 2-digit level of ISIC Rev. 3, which gives 37 manufacturing industries. To deal with the non-manufacturing industries, we group them into a "residual" sector which we denote by  $S$ . To incorporate this sector in the analysis, we set

$$\alpha_S = 1 - \sum_{j \in M} \alpha_j, \quad (81)$$

where  $M$  is the set of manufacturing sectors. Because in our theory this sector is not engaged in foreign sourcing<sup>76</sup>, we set

$$\Lambda_S = 0.$$

The input-output structure of sector  $S$  can be recovered from the input-output tables. In particular, we set

$$\zeta_j^S \equiv \frac{\sum_{n=1}^{NM} \text{Intermediate supply from industry } j \text{ to industry } n}{\sum_{n=1}^{NM} \text{Intermediate consumption at final prices to industry } n},$$

where  $NM$  is the set of non-manufacturing sectors. To measure the materials coefficient in the production of sector  $S$ , we employ the Input-Output Matrix for the non-manufacturing sectors. As we observe value added and intermediary spending for each sector, we set

$$\gamma_S = \frac{\sum_{n=1}^{NM} X_n}{\sum_{n=1}^{NM} (X_n + VA_n)}, \quad (82)$$

where  $X_n$  denotes total intermediary spending by sector  $n$ . Table 16 summarizes how  $[\alpha_s]$  and  $[\gamma_s]$  are computed. The input-output matrix  $\Xi$  used in our empirical analysis is contained in Table 15.

<sup>76</sup>Note that this sector may nevertheless benefit from input trade if it sources output from the manufacturing industries.

Sector	10-14	15-16	17-19	20	21-22	24	25	26	27	28	29	30	31	32	33	34	35	36-37	S
10-14	8.69	0.12	0	0.02	0.41	1.8	0.26	9.83	4.85	0.36	0.12	0.05	0.16	0.07	0.12	0	0	0.82	0.86
15-16	0.48	21.33	2.27	0.1	0.51	1.97	0.24	0.06	0.17	0.17	0.12	0.1	0.12	0.18	0.18	0.04	0.04	0.48	2.95
17-19	0.26	0.11	46.79	0.08	0.65	0.8	1.39	0.44	0.27	0.17	0.34	0.28	0.42	0.63	0.87	1.4	0.48	5.95	0.38
20	1.33	0.38	0.13	30.47	1.07	0.17	0.38	2.06	0.33	0.39	0.29	0.09	0.23	0.32	0.32	0.29	0.41	15.9	0.59
21-22	1.44	2.29	1.73	1.45	44.73	3.02	2.27	2.82	0.6	0.63	1.01	0.82	1.2	1.37	2.11	0.46	0.52	2.74	3.02
24	4.53	1.25	8.25	1.69	5.03	39.28	40.03	3.31	2.92	3.02	1.78	0.57	4.42	0.84	1.17	1.93	1.03	4.17	1.93
25	1.68	2.46	2.36	0.65	1.67	3.03	15.72	1.44	0.66	2.49	3.65	1.29	6.58	5.07	3.23	6.57	1.64	4.93	0.95
26	8.53	0.79	0.19	0.81	0.21	0.81	0.66	21.53	0.88	0.9	0.67	0.11	0.81	1.5	2.08	1.23	0.36	1.86	1.79
27	0.62	0.09	0.4	0.64	0.8	0.76	1.67	3.24	41.61	24.72	9	0.48	11.46	2.5	5.66	8.44	2.28	9.25	0.35
28	5.95	1.39	1.16	3.51	0.98	2.13	1.78	2.26	6.92	28.04	15.87	0.75	13.11	5.97	8.31	8.02	4.84	4.72	1.38
29	20.33	0.79	1.66	3.31	0.87	0.78	2.79	2.6	2	4.04	19.27	0.64	2.28	1.99	3.37	4.21	3.33	3.01	1.51
30	0	0.02	0.04	0.36	0.08	0.02	0.08	0.36	0.04	0.24	0.48	37.61	0.35	1.27	2.02	0	0.07	0.17	0.34
21	0.51	0.14	0.08	0.28	0.35	0.34	0.21	0.79	1.07	2.24	4.43	3.93	16.03	6.83	2.84	3.07	1.05	1.39	1.12
32	0.9	0.06	0.07	0.09	0.15	0.13	0.39	0.37	0.34	0.59	3.81	12.59	11	30.3	11.86	1.98	4.26	1.5	0.7
33	0.07	0.01	0.08	0.03	0.12	0.12	0.22	0.57	0.39	0.61	2.29	2.59	2.72	8.55	19.31	1.52	8.74	0.42	0.6
34	0.68	0.09	0.12	0.22	0.12	0.07	0.07	0.47	0.21	0.25	0.69	0.13	0.27	0.22	0.24	35.3	0.14	0.42	0.82
35	0.15	0.01	0.04	0.04	0.05	0.04	0.05	0.06	0.05	0.11	1.31	0.02	0.03	0.03	0.05	0.11	46.37	0.05	1.12
36-37	0.2	0.17	0.59	0.42	0.8	0.18	0.23	0.43	1.34	0.98	0.76	0.31	0.61	0.34	0.52	1.94	0.37	6.85	0.45
S	43.64	68.48	34.03	55.82	41.4	44.54	31.56	47.37	35.35	30.04	34.13	37.63	28.21	32.02	35.73	23.46	24.09	35.37	79.14

Notes: The table contains the French input-output matrix used in our empirical work. We report numbers in percentage terms. Sectors are classified at the 2-digit-level according to ISIC Rev. 3. The non-manufacturing sector  $S$  is constructed as explained in the main text and Table 16.

Table 15: Input-Output Linkages: E

Industry	ISIC	$\phi_k$	$\phi_l$	$\gamma$
Mining	10-14	0.374*** (0.039)	0.293*** (0.017)	0.333*** (0.043)
Food, tobacco, beverages	15-16	0.098*** (0.004)	0.177*** (0.003)	0.725*** (0.006)
Textiles and leather	17-19	0.081*** (0.003)	0.293*** (0.009)	0.626*** (0.012)
Wood and wood products	20	0.113*** (0.004)	0.285*** (0.006)	0.602*** (0.006)
Paper, printing, publishing	21-22	0.134*** (0.007)	0.362*** (0.011)	0.504*** (0.011)
Chemicals	24	0.124*** (0.008)	0.204*** (0.01)	0.671*** (0.014)
Rubber and plastics products	25	0.124*** (0.005)	0.289*** (0.007)	0.587*** (0.011)
Non-metallic mineral products	26	0.178*** (0.01)	0.294*** (0.012)	0.529*** (0.015)
Basic metals	27	0.124*** (0.01)	0.202*** (0.015)	0.674*** (0.021)
Metal products (ex machinery and equipment)	28	0.108*** (0.002)	0.412*** (0.008)	0.479*** (0.009)
Machinery and equipment	29	0.071*** (0.003)	0.313*** (0.015)	0.616*** (0.018)
Office and computing machinery	30	0.037*** (0.012)	0.150*** (0.032)	0.813*** (0.04)
Electrical machinery	31	0.096*** (0.008)	0.306*** (0.011)	0.598*** (0.014)
Radio and communication	32	0.055*** (0.006)	0.322*** (0.048)	0.624*** (0.052)
Medical and optical instruments	33	0.071*** (0.004)	0.435*** (0.026)	0.494*** (0.029)
Motor vehicles, trailers	34	0.106*** (0.009)	0.135*** (0.016)	0.759*** (0.014)
Transport equipment	35	0.152*** (0.019)	0.499*** (0.03)	0.349*** (0.044)
Manufacturing, recycling	36-37	0.084*** (0.003)	0.283*** (0.009)	0.633*** (0.012)

Notes: The table contains the production function parameters based on observed factor shares. See Section 3.1 in the main text for details.

Table 16: Production Function Coefficient Estimates, by 2-digit Sector: Factor Shares

## A.2 Estimating the Parameters of the Production Function

We report the results of estimating the production function parameters using our different approaches. In Table 16, we report the results of the factor share approach. Note that this method imposes the assumption of constant returns, so that  $\phi_{ks} + \phi_{ls} + \gamma_s = 1$ . Table 17 reports the results based on proxy methods akin to Levinsohn and Petrin (2012) and Wooldridge (2009). We do not impose constant returns to scale for these approaches. We assume that labor is a dynamic input, which seems plausible given the stringent hiring and firing regulations of the French economy. Note that we do not include firms' domestic share in material spending in the production function as we estimate  $\varepsilon$  in the second stage. Finally, Table 18 contains the results of the integrated GMM approach, where we treat the domestic expenditure share as a distinct input and estimate the parameter vector  $(\phi_{ks}, \phi_{ls}, \gamma_s, \varepsilon_s)$  in one step.

## A.3 Bootstrap Procedure

We sample firms from the empirical distribution with replacement to construct 200 replicates of our micro-data. For each of these samples, we re-calculate  $\sigma_s$  and re-estimate  $\varepsilon$  and  $[\gamma_s]$  following the factor shares approach explained in Section 3.1 and then re-calculate  $[\Lambda_s]$  and  $[s_D^{Agg}]$ . Figure 6 depicts the bootstrap distributions of these variables. For the three sector-level variables, we report the distribution of the sectoral averages, e.g. the upper right panel displays the distribution of  $\frac{1}{S} \sum_{s=1}^S \gamma_s$ . While the variation in  $\gamma$  and  $s_D^{Agg}$  is relatively modest, there is a quite a bit of

Industry	ISIC	$\phi_k$		$\phi_l$		$\gamma$	
Mining	10-14	0.647***	(0.101)	0.626***	(0.087)	0.295**	(0.139)
Food, tobacco, beverages	15-16	0.174***	(0.010)	0.274***	(0.009)	0.538***	(0.060)
Textiles and leather	17-19	0.216***	(0.026)	0.513***	(0.037)	0.481***	(0.096)
Wood and wood products	20	0.138***	(0.023)	0.414***	(0.024)	0.521***	(0.058)
Paper, printing, publishing	21-22	0.061***	(0.022)	0.717***	(0.033)	0.600***	(0.099)
Chemicals	24	0.027	(0.081)	0.142	(0.134)	1.304***	(0.336)
Rubber and plastics products	25	0.148***	(0.031)	0.536***	(0.050)	0.357***	(0.115)
Non-metallic mineral products	26	0.221***	(0.037)	0.539***	(0.037)	0.357***	(0.119)
Basic metals	27	0.104	(0.096)	0.381***	(0.087)	0.481*	(0.263)
Metal products (ex machinery and equipment)	28	0.252***	(0.013)	0.655***	(0.016)	0.231***	(0.036)
Machinery and equipment	29	0.186***	(0.018)	0.563***	(0.025)	0.393***	(0.066)
Office and computing machinery	30	0.170**	(0.081)	0.574***	(0.110)	0.104	(0.210)
Electrical machinery	31	0.144***	(0.031)	0.448***	(0.044)	0.449***	(0.123)
Radio and communication	32	0.123**	(0.057)	0.565***	(0.114)	0.568***	(0.207)
Medical and optical instruments	33	0.231***	(0.021)	0.501***	(0.020)	0.421***	(0.073)
Motor vehicles, trailers	34	0.082	(0.091)	0.316**	(0.127)	0.765**	(0.355)
Transport equipment	35	0.194***	(0.073)	0.686***	(0.089)	0.997***	(0.256)
Manufacturing, recycling	36-37	0.242***	(0.018)	0.472***	(0.016)	0.430***	(0.055)

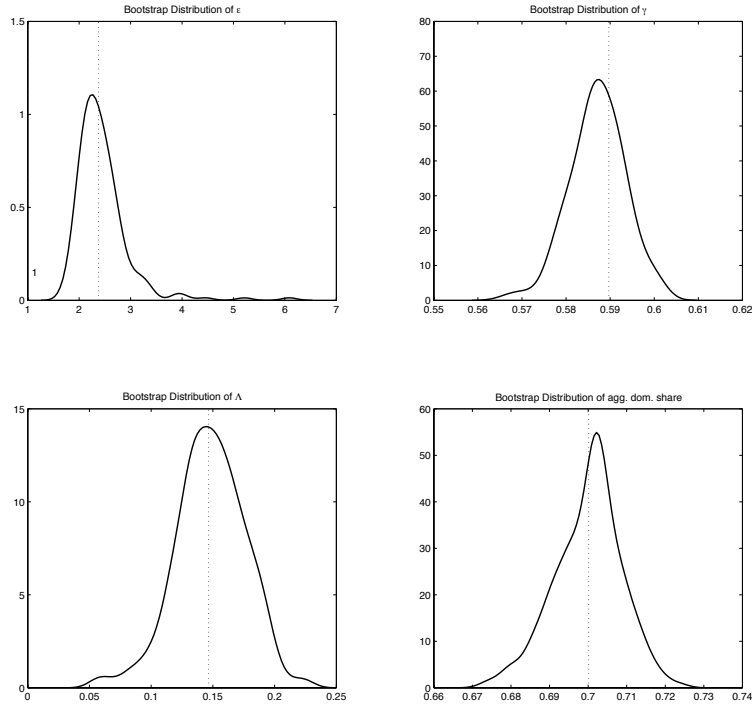
Notes: The table contains the production function parameters based on the GMM procedure by Levinsohn and Petrin (2012) and Wooldridge (2009). See Section 3.1 in the main text for details.

Table 17: Production Function Coefficient Estimates, by 2-digit Sector: GMM

Industry	ISIC	$\phi_k$		$\phi_l$		$\gamma$		$\varepsilon$	
Mining	10-14	0.679***	(0.100)	0.617***	(0.088)	0.257*	(0.148)	0.450	(0.451)
Food, tobacco, beverages	15-16	0.173***	(0.010)	0.278***	(0.009)	0.512***	(0.063)	1.976***	(0.166)
Textiles and leather	17-19	0.247***	(0.029)	0.555***	(0.035)	0.354***	(0.101)	2.279***	(0.743)
Wood and wood products	20	0.172***	(0.025)	0.437***	(0.025)	0.441***	(0.061)	2.548***	(0.324)
Paper, printing, publishing	21-22	0.089***	(0.022)	0.726***	(0.032)	0.465***	(0.099)	2.189***	(0.383)
Chemicals	24	0.072	(0.056)	0.250***	(0.084)	1.064***	(0.229)	-6.586	(6.818)
Rubber and plastics products	25	0.163***	(0.032)	0.584***	(0.055)	0.253**	(0.128)	2.442**	(1.103)
Non-metallic mineral products	26	0.221***	(0.036)	0.526***	(0.036)	0.354***	(0.121)	1.869***	(0.358)
Basic metals	27	0.151	(0.120)	0.540***	(0.177)	-0.084	(0.583)	0.892	(0.692)
Metal products (ex machinery and equipment)	28	0.254***	(0.014)	0.667***	(0.016)	0.191***	(0.036)	1.368***	(0.0796)
Machinery and equipment	29	0.183***	(0.018)	0.553***	(0.024)	0.381***	(0.065)	2.191***	(0.279)
Office and computing machinery	30	0.132	(0.087)	0.451***	(0.110)	0.126	(0.222)	2.358	(3.485)
Electrical machinery	31	0.138***	(0.028)	0.460***	(0.037)	0.428***	(0.110)	2.806***	(0.863)
Radio and communication	32	0.132*	(0.069)	0.614***	(0.144)	0.442*	(0.264)	3.147	(2.733)
Medical and optical instruments	33	0.237***	(0.021)	0.498***	(0.020)	0.372***	(0.073)	2.062***	(0.295)
Motor vehicles, trailers	34	0.089	(0.087)	0.354***	(0.120)	0.706**	(0.344)	3.741*	(2.088)
Transport equipment	35	0.170**	(0.081)	0.664***	(0.094)	0.955***	(0.282)	4.482	(4.371)
Manufacturing, recycling	36-37	0.263***	(0.019)	0.468***	(0.017)	0.361***	(0.059)	1.765***	(0.183)

Notes: The table contains the production function parameters based on the GMM procedure by Levinsohn and Petrin (2012) and Wooldridge (2009). See Section 3.1 in the main text for details.

Table 18: Production Function Coefficient Estimates, by 2-digit Sector: GMM with  $s_D$  as input



Notes: The upper left panel contains the bootstrap distribution of  $\varepsilon$ . The remaining panels depict the bootstrap distributions of  $\frac{1}{S} \sum_{s=1}^S \gamma_s$ ,  $\frac{1}{S} \sum_{s=1}^S \Lambda_s$  and  $\frac{1}{S} \sum_{s=1}^S s_{D_s}^{Agg}$ . The point estimates used in the main analysis are reported as vertical lines.

Figure 6: Bootstrap Distribution of Structural Parameters and Direct Price Reductions

uncertainty regarding  $\varepsilon$ . This is consistent with the non-negligible standard errors reported in Table 2. We conclude that it is the variation in  $\varepsilon$  which induces most of the variation in  $\Lambda$  and therefore in the consumer price gains from input trade reported in Tables 5-6 and shown in Figure 5.

#### A.4 General equilibrium and Welfare in the Model of Section 4.1

Consider the setup of Section 4.1. We now consider the aggregate allocations in this economy. An equilibrium has the usual definition:

**Definition 1.** *An equilibrium is a set of prices  $w, [p_i]$ , labor demands for production and fixed costs  $[l_i, l_i^F]$ , differentiated product quantities, consumption levels and foreign demands  $[y_i, c_i, y_i^{ROW}]$ , domestic and international input demands by local firms  $[y_{vi}], [z_{ci}]$  and sourcing strategies  $[n_i]$  such that:*

1. *Firms maximize profits given by (36)-(37),*
2. *Consumers maximize utility given by (12) and (13) subject to their budget constraint*

$$\int_i p_i c_i di = wL + \int_i \pi_i di, \quad (83)$$

3. Trade is balanced (39),

4. Labor and good markets clear

$$\begin{aligned} L &= \int_i (l_i + l_i^F) di \\ y_i &= c_i + y_i^{ROW} + \int_\nu y_{vi} dv. \end{aligned}$$

We first characterize the general equilibrium in a multi-sector version of the economy of Section 4.1. In particular, we consider the multi-sector structure of Section 2.2. We derive a generalization of (40). We do not impose any assumptions on how firms' determine their extensive margin. That is, we allow for an arbitrary mapping  $l_{\Sigma_i}$  which gives the labor resources that firm  $i$  needs to spend in order to attain the sourcing strategy  $\Sigma_i$ . We assume that trade is balanced and that the value of exports in sector  $s$  is given by  $\alpha_s^{ROW} \times IM$ , where  $IM$  denotes the value of total spending on imported inputs.

**Proposition 4.** *Let  $W, I$  and  $S$  denote welfare, consumer income and total spending, respectively. Then, the change in welfare relative to input autarky is given by*

$$\frac{W}{W^{Aut}} = \frac{I}{I^{Aut}} \times \frac{P^{Aut}}{P},$$

where  $I$  and  $I^{Aut}$  are given by

$$I = L + \sum_{s=1}^S S_s / \sigma_s - \sum_{s=1}^S \left( \int_0^{N_s} l_{\Sigma_i} di \right), \quad (84)$$

$$I^{Aut} = L + \sum_{s=1}^S S_s^{Aut} / \sigma_s, \quad (85)$$

and  $[S_j]$  and  $[S_j^{Aut}]$  solve

$$S_s = \alpha_s \left( L - \sum_{j=1}^S \left( \int_0^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^S \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^S [\alpha_s^{ROW} - \zeta_s^j] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{D_i}) \omega_i di, \quad (86)$$

and

$$S_s^{Aut} = \alpha_s \left( L + \sum_{j=1}^S \frac{1 + \zeta_s^j \gamma_j (\sigma_j - 1) / \alpha_s}{\sigma_j} S_j^{Aut} \right), \quad (87)$$

Furthermore,  $G = \frac{P^{Aut}}{P}$  is given in Proposition 2.

*Proof.* As labor is the only factor of production, consumer welfare is given by real income  $W = I/P$ ,

consumer income is given by

$$I = L + \sum_{s=1}^S \left( \int_0^{N_s} \pi_i di \right).$$

Note that  $L$  represents total labor income and  $\pi_i$  denotes firm  $i$ 's profits. To derive  $\pi_i$ , recall that firms in sector  $s$  have a mark-up of  $\sigma_s/(\sigma_s - 1)$  so that variable profits gross of any extensive margin resource loss are given by

$$\pi_i^V = (p_i - u_i) y_i = p_i y_i / \sigma_s. \quad (88)$$

Total revenue for firm  $i$  is given by

$$p_i y_i = \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} S_s, \quad (89)$$

where  $P_s$  is the consumer price index for sector  $s$  and  $S_s$  denotes total spending for sector  $s$  goods. Hence,

$$\pi_i = p_i y_i / \sigma_s - l_{\Sigma_i} = \frac{1}{\sigma_s} \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} S_s - l_{\Sigma_i},$$

so that

$$I = L + \sum_{s=1}^S \frac{1}{\sigma_s} S_s - \sum_{s=1}^S \left( \int_0^{N_s} l_{\Sigma_i} di \right). \quad (90)$$

Hence, given  $[S_s]$  and  $[l_{\Sigma_i}]$ , total income  $I$  is fully determined. Now consider  $[S_s]_s$ . Note that

$$S_s = S_s^C + S_s^X + S_s^{ROW}, \quad (91)$$

where  $S_s^C, S_s^X$  and  $S_s^{ROW}$  denote total spending by consumers, intermediary producers and the rest of the world, respectively. For our economy, we have that  $S_s^C = \alpha_s I$  and  $S_s^{ROW} = \alpha_s^{ROW} Im$  as consumers spend a fraction  $\alpha_s$  of their income  $I$  on sector  $s$  products and balanced trade requires that total spending by the rest of the world is equal to the value of imports  $Im$ , a fraction  $\alpha_s^{ROW}$  of which is spent on sector  $s$  products. To derive  $S_s^X$ , let total domestic intermediary purchases in sector  $j$  be given by  $X_j$ . Then

$$S_s^X = \sum_{j=1}^S \zeta_s^j X_j. \quad (92)$$

Letting  $m_i$  be total material spending by firm  $i$  and  $s_i$  be total spending by firm  $i$ , we know that

$$\begin{aligned} X_j &= \int_0^{N_j} s_{Di} m_i di = \int_0^{N_j} s_{Di} \gamma_j s_i di = \int_0^{N_j} s_{Di} \gamma_j \frac{\sigma_j - 1}{\sigma_j} p_i y_i di \\ &= \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di, \end{aligned} \quad (93)$$

where we used that firms in sector  $j$  spend a fraction  $\gamma_j$  of their total input spending  $s_i$  on materials and that total spending  $s_i$  accounts for a fraction  $(\sigma_j - 1)/\sigma_j$  of revenue. Hence, (92) and (93) imply



that

$$S_s^X = \sum_{j=1}^S \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} s_{Di} \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} di. \quad (94)$$

Similarly, total import spending is equal to

$$\begin{aligned} Im &= \sum_{j=1}^S Im_j = \sum_{j=1}^S \int_0^{N_j} (1 - s_{Di}) m_i di \\ &= \sum_{j=1}^S \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_{i=0}^{N_j} (1 - s_{D,i}) \left( \frac{p_i}{P_s} \right)^{1-\sigma_s} di. \end{aligned} \quad (95)$$

Hence (94) and (95) imply that

$$\begin{aligned} S_s &= \alpha_s I + \alpha_s^{ROW} \left( \sum_{j=1}^S \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di \right) + \sum_{j=1}^S \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} s_{Di} \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di \\ &= \alpha_s I + \sum_{j=1}^S \zeta_s^j \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j + \sum_{j=1}^S [\alpha_s^{ROW} - \zeta_s^j] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_i^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di. \end{aligned}$$

Using (90), we get that

$$S_s = \alpha_s \left( L - \sum_{j=1}^S \left( \int_i^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^S \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^S [\alpha_s^{ROW} - \zeta_s^j] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_i^{N_j} (1 - s_{Di}) \left( \frac{p_i}{P_j} \right)^{1-\sigma_j} di.$$

Now note that

$$\frac{va_i}{\int_0^{N_j} va_i di} = \frac{p_i y_i}{\int_0^{N_j} p_i y_i di} = \frac{(p_i/P_s)^{1-\sigma_s} S_s}{\int_0^{N_j} (p_i/P_s)^{1-\sigma_s} S_s di} = \left( \frac{p_i}{P_s} \right)^{1-\sigma_s}.$$

Hence,

$$S_s = \alpha_s \left( L - \sum_{j=1}^S \left( \int_0^{N_j} l_{\Sigma_i} di \right) + \sum_{j=1}^S \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j \right) + \sum_{j=1}^S [\alpha_s^{ROW} - \zeta_s^j] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \omega_i di \quad (96)$$

where  $\omega_i = \frac{va_i}{\int_0^{N_j} va_i di}$ . Given  $L^{NET} = L - \sum_{j=1}^S \left( \int_0^{N_j} l_{\Sigma_i} di \right)$ , (96) are  $S$  equations in  $S$  unknowns  $S_s$ , which we can easily solve. Now consider the case of autarky. There we have  $l_{\Sigma_i} = 0$  and  $s_{Di} = 1$ . Hence, (96) yields

$$S_s^{Aut} = \alpha_s \left( L + \sum_{j=1}^S \frac{1 + \frac{\zeta_s^j}{\alpha_s} \gamma_j (\sigma_j - 1)}{\sigma_j} S_j^{Aut} \right).$$

In the case of a single sector (i.e.  $S = 1$ ) it has to be the case that

$$\alpha_S = \alpha_S^{ROW} = \zeta_S^S = 1.$$

Hence,

$$S^{Aut} = L + \frac{1 + \gamma(\sigma - 1)}{\sigma} S^{Aut} = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} L.$$

Substituting this in (90) yields

$$I^{Aut} = L + \frac{1}{\sigma}S = \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)}L.$$

Similarly, we get from (96) that

$$\sum_{j=1}^S [\alpha_s^{ROW} - \zeta_s^j] \gamma_j \frac{\sigma_j - 1}{\sigma_j} S_j \int_0^{N_j} (1 - s_{Di}) \omega_i di = 0$$

so that

$$S = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \left( L - \left( \int_i^N l_{\Sigma_i} di \right) \right) \quad (97)$$

$$I = \frac{1 + (1 - \gamma)(\sigma - 1)}{(1 - \gamma)(\sigma - 1)} \left( L - \left( \int_i^N l_{\Sigma_i} di \right) \right). \quad (98)$$

This implies directly (40). This concludes the proof of Proposition 4.  $\square$

## A.5 Calibrating the Model of Section 4

We adopt a solution algorithm that allows us to bypass the computation of the general equilibrium variables within the calibration. Intuitively, we work with a normalized version of fixed costs, where these are scaled by an appropriate transformation of the general equilibrium variables. Because the equilibrium variables depend on firms' import behavior only through the domestic shares, which are itself a calibration target, we can compute them *after* the calibration. That is, we can first ensure that the moments of the joint distribution of value added and domestic shares are matched<sup>77</sup>, and then back out the underlying general equilibrium variables required to compute welfare. We also show that the parameter  $z$  is not required for the calibration.

We first start with three aggregate variables, which are determined in equilibrium. In the single-sector version of the model, characterized in Section A.4 in the Online Appendix, we have that aggregate spending  $S$  and the price level (which is also equal to the price of domestic varieties) is given by

$$S = \frac{\sigma}{(1 - \gamma)(\sigma - 1)} \left( L - \left( \int_i^N l_{\Sigma_i} di \right) \right) \quad (99)$$

$$P = \left( \frac{\sigma}{\sigma - 1} \left( \frac{1}{\gamma} \right)^\gamma \left( \frac{1}{1 - \gamma} \right)^{1 - \gamma} \left( \frac{1}{qD} \right)^\gamma \mathcal{Y} \right)^{\frac{1}{1 - \gamma}}, \quad (100)$$

where

$$\mathcal{Y} = \left( \int_{i=0}^N \left( \frac{1}{\tilde{\varphi}_i} (s_{D,i})^{\gamma/(\varepsilon - 1)} \right)^{1 - \sigma} di \right)^{\frac{1}{1 - \sigma}}. \quad (101)$$

---

<sup>77</sup>For this step, it is important that the dispersion and correlation moments are in logs. See below.

We start by noting that the firm's optimality conditions from the profit maximization problem, contained in Section 7.6, can be expressed in terms of  $s_D$  instead of  $n$ . To see this, note that (8) and (35) imply

$$n^{\eta(\varepsilon-1)} = \left( \frac{1-s_D}{s_D} \right) \left( \frac{\beta}{1-\beta} \right)^\varepsilon z^{\varepsilon-1} \left( \frac{q_D}{p_D} \right)^{\varepsilon-1}. \quad (102)$$

Substituting (102) into the firm's first order condition (73), we obtain

$$s_D^{\frac{1-\gamma(\sigma-1)\eta}{(\varepsilon-1)\eta}} (1-s_D)^{1-\frac{1}{\eta(\varepsilon-1)}} = \left( \frac{\beta}{1-\beta} \right)^{\frac{\varepsilon-1}{\eta}} \frac{\tilde{f}}{\tilde{\varphi}^{\sigma-1}}, \quad (103)$$

where

$$\tilde{f} \equiv f \times (zq_D)^{1/\eta} \frac{1}{\eta\gamma(\sigma-1)} \frac{1}{\Theta} \times \frac{1}{P^{1/\eta\Gamma}}, \quad (104)$$

where

$$\Theta = \frac{1}{\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \left( \left( \frac{1}{1-\gamma} \right)^{1-\gamma} \left( \frac{1}{\gamma} \left( \frac{1}{q_D} \right) \right)^\gamma \right)^{1-\sigma}, \quad (105)$$

$$\Gamma = \frac{S}{P^{(1-\gamma)(1-\sigma)}}. \quad (106)$$

Similarly, (102) and the import status condition (79) imply that the firm is an importer as long as

$$\left[ s_D^{\frac{-\gamma(\sigma-1)}{\varepsilon-1}} - 1 \right] \tilde{\varphi}^{\sigma-1} - \left( \frac{1-s_D}{s_D} \right)^{\frac{1}{\eta(\varepsilon-1)}} \gamma(\sigma-1)\eta \left( \frac{\beta}{1-\beta} \right)^{\frac{\varepsilon-1}{\eta}} \tilde{f} - \tilde{f}_I > 0, \quad (107)$$

where

$$\tilde{f}_I \equiv \frac{1}{\Gamma} \frac{1}{\Theta} \times f_I. \quad (108)$$

(103) and (107) show that we can solve for firms' optimal domestic share and import status with knowledge of  $\tilde{\varphi}^{\sigma-1}$ ,  $\tilde{f}$  and  $\tilde{f}_I$  only. Thus, we can work with the joint distribution of  $(\varphi, \tilde{f})$  to match the moments of the joint distribution of domestic shares and value added. We can then back out the exogenous component of fixed costs  $f_I$  and  $f$  from  $\tilde{f}_I$  and  $\tilde{f}$  using the equilibrium variables  $S$  and  $P$  and (106).

To solve for  $S$ , we require the aggregate resource loss of fixed costs (see (99)). To do so, note that

$$\begin{aligned} l_{\Sigma_i} = l_i(s_{Di}) &= f_i \times \left( \frac{s_{Di}}{1-s_{Di}} \right)^{\frac{1}{\eta(1-\varepsilon)}} \left( \frac{1}{P} \right)^{1/\eta} (q_D z)^{1/\eta} \left( \frac{\beta_i}{1-\beta_i} \right)^{\frac{\varepsilon-1}{\eta}} + f_I \\ &= \Gamma\Theta \left\{ \eta\gamma(\sigma-1) \times \tilde{f}_i \times \left( \frac{s_{Di}}{1-s_{Di}} \right)^{\frac{1}{\eta(1-\varepsilon)}} \left( \frac{\beta_i}{1-\beta_i} \right)^{\frac{\varepsilon-1}{\eta}} + \tilde{f}_I \right\}. \end{aligned}$$

Hence,

$$\int_i^N l_{\Sigma_i} di = \Gamma\Theta \left\{ \eta\gamma(\sigma-1) \times \int_i^N \tilde{f}_i \left( \frac{s_{Di}}{1-s_{Di}} \right)^{\frac{1}{\eta(1-\varepsilon)}} \left( \frac{\beta_i}{1-\beta_i} \right)^{\frac{\varepsilon-1}{\eta}} di + \int_i^N \tilde{f}_I 1[s_{Di}] di \right\}. \quad (109)$$

The key is now to argue that  $\Gamma$  is known given the calibration. If so, we can calculate  $\int_i^N l_{\Sigma_i} di$  from (109) given the calibrated  $\tilde{f}$  and  $\tilde{f}_I$  and parameters, as

$$\int_i^N l_{\Sigma_i} di = \Gamma \times \Theta \times \Delta,$$

where

$$\Delta \equiv \eta\gamma(\sigma - 1) \times \int_i^N \tilde{f}_i \left( \frac{s_{Di}}{1 - s_{Di}} \right)^{\frac{1}{\eta(1-\varepsilon)}} \left( \frac{\beta_i}{1 - \beta_i} \right)^{\frac{\varepsilon-1}{\varepsilon} \frac{1}{\eta}} di + \int_i^N \tilde{f}_I 1 [s_{Di}] di. \quad (110)$$

Recall that (106) and (99) imply that

$$\begin{aligned} \Gamma &= \frac{S}{P^{(1-\gamma)(1-\sigma)}} = \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \left( L - \left( \int_i^N l_{\Sigma_i} di \right) \right) \\ &= \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} L - \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \Gamma \Theta \Delta. \end{aligned}$$

Solving for  $\Gamma$  yields

$$\Gamma = \frac{\frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)}}{1 + \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \Theta \Delta} L. \quad (111)$$

As  $L$  is a normalization (see below), (111) shows that  $\Gamma$  is fully determined as  $P$  can be evaluated from the calibrated data on domestic shares (see (100) and (101)). Hence,

$$\int_i^N l_{\Sigma_i} di = \Gamma \Theta \Delta = \frac{\frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \Theta \Delta}{1 + \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \Theta \Delta} L.$$

This implies that

$$\frac{L - \int_i^N l_{\Sigma_i} di}{L} = \frac{1}{1 + \frac{1}{P^{(1-\gamma)(1-\sigma)}} \frac{\sigma}{(1-\gamma)(\sigma-1)} \Theta \Delta}, \quad (112)$$

so that  $L$  is indeed a normalization. Finally we only have to show that (112) does not depend on  $q_D$ , even though  $\Theta$  does (see (105)). However, it can easily be shown that

$$\Theta P^{(1-\gamma)(\sigma-1)} = \gamma^{\sigma-1} \frac{1}{\sigma}. \quad (113)$$

Hence, the quality of domestic varieties  $q_D$  and the foreign price level  $z$  can be normalized for the calibration.

The five models we consider fit in this framework as follows:

1. The *aggregate model* assumes that  $\beta_i = \beta$  and  $f_i = f_I = 0$ . Hence,  $\int_i^N l_{\Sigma_i} di = 0$  and  $s_{Di} = s_D$  can be solved from (102) using that  $n = 1$  (as all firms are importers and import from every country). The level of  $\beta$  is chosen to match the aggregate domestic share. The dispersion in productivity  $\sigma_\varphi$  is chosen to match the dispersion in value added.
2. The *homogenous bias model* assumes that  $\beta_i = \beta$  and  $f_i = 0 < f_I$ . Hence, conditional on

importing, we have that  $s_{Di} = s_D$ , which can be solved from (102) using that  $n = 1$ . The required level  $\tilde{f}_I$  in (107) is chosen to match the share of importers. Given a distribution of productivity  $[\tilde{\varphi}_i]_i$ , we can then calculate  $\Delta$  from (110),  $P$  from (100) and (101) and hence  $\Gamma$  from (111). This is sufficient to calculate welfare using (112) and  $P^{Aut}/P$ .

3. The *heterogeneous bias model* assumes that  $\beta_i$  varies across firms and  $f_i = 0 < f_I$ . As for the case with fixed costs, it is useful to consider a scaled version of the home-bias  $\tilde{\beta}_i = \frac{\beta_i}{1-\beta_i}$ . In particular, (102) shows that  $s_D$  only depends on  $\beta^* = \left(\tilde{\beta}\right)^\varepsilon \left(\frac{1}{p}\right)^{\varepsilon-1}$  (again, we have  $n = 1$  as there are no fixed costs per country). Hence, we draw  $(\tilde{\varphi}, \beta^*)$  from a joint log-normal distribution. Using (102), this generates a joint distribution of  $(\tilde{\varphi}_i, s_{Di})$ . We can then calibrate  $\tilde{f}_I$  from (107) to match the share of importers. Like for the case of the homogenous bias model, we can then use (110),  $P$  and (111) to compute all equilibrium objects.
4. For the *heterogenous fixed cost model*, we draw  $(\tilde{\varphi}_i, \tilde{f}_i)$  from a joint log-normal distribution. Using the (103), this implies a joint distribution of  $(\tilde{\varphi}_i, s_{Di})$ . We can then calibrate  $\tilde{f}_I$  from (107) to match the share of importers. As above, we can then use (110),  $P$  and (111) to compute all equilibrium objects.
5. The *homogenous fixed cost model*, is a special case of the heterogenous fixed cost model where  $\tilde{f}_i = \tilde{f}$ . Hence, the procedure is exactly the same given a marginal distribution for  $\tilde{\varphi}_i$ .