# Errata For "Solutions Manual for *Introduction to Modern Economic Growth*: Student Edition"

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This document contains an incomplete list of typos and errors in the student edition of the solutions manual for *Introduction to Modern Economic Growth*. We would appreciate it if readers could e-mail us concerning errors, corrections or alternative solutions, which we will include in the next update of this document. Our present e-mail addresses are as follows:

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## Chapter 2: The Solow Growth Model

### Exercise 2.11

In some of the earlier prints of the manual, seven instances of the phrase "capital-labor ratio" are missing from various places on pages 1 and 2. In case your copy has this error, please note the following corrections.

- **Page 1, immediately after Eq.** (S2.1)**:** "Let $k(t) = K(t)/L(t)$ denote the ." should be "Let $k(t) = K(t)/L(t)$ denote the capital-labor ratio."
- **Page 1, immediately before the first displayed equation after Eq.** (S2.1)**:** "the evolution of the is given by" should be "the evolution of the capital-labor ratio is given by".
- **Page 1, immediately before Eq.** (S2.2)**:** "the unique positive steady state can be solved as" should be "the unique positive steady state capital-labor ratio can be solved as".
- **Page 1, the third to last sentence of Part (a):** "the converges to the unique positive steady state level $k^*$" should be "the capital-labor ratio converges to the unique positive steady state level $k^*$".
- **Page 1, the last sentence of Part (a):** "there is a unique steady state with a positive despite the fact that" should be "there is a unique steady state with a positive capital-labor ratio despite the fact that".
- **Page 1, immediately before Eq.** (S2.4)**:** "the evolves according to" should be "the capital-labor ratio evolves according to".
- **Page 2, first full paragraph, third sentence:** "the only steady state is $k^* = 0$" should be "the only steady state capital-labor ratio is $k^* = 0$".

# Chapter 6: Infinite-Horizon Optimization and Dynamic Programming

## Exercise 6.8, Part (c).

The original solution is incorrect because it does not take the non-negativity constraints for consumption into account. We are very grateful to Guillaume Rocheteau for pointing this out to us. That our original solution can involve consumption being negative is most clearly seen in (S6.9), were we derived the consumption rule as

$$c(k) = \frac{A^2\beta - 1}{A\beta}k - \frac{\beta A - 1}{aA\beta(A-1)}.$$

For $A\beta > 1$, consumption will be negative for $k$ small enough. So let us assume that $A\beta > 1$.

The correct solution is as follows. The recursive formulation of the problem is given by

$$\begin{aligned} V(k) &= \max_{k'} \left\{ Ak - k' - \frac{a}{2}(Ak - k')^2 + \beta V(k') \right\} \\ & \quad \text{s.t. } Ak - k' \geq 0. \end{aligned} \tag{1}$$

Letting $\lambda(k) \geq 0$ be the multiplier on the constraint (1), the FOC for this problem is given by

$$a(Ak - k') + \beta V'(k') = 1 + \lambda(k),$$

with the complementary slackness condition

$$(Ak - k')\lambda(k) = 0.$$

The Envelope condition is given by

$$V'(k) = A(1 - a(Ak - k')) + \lambda(k).$$

Note first that

$$\lambda(k) = 0 \Rightarrow \lambda\left(\tilde{k}\right) = 0 \text{ for all } \tilde{k} \geq k,$$

i.e. if the consumption choice is interior for a capital-level of $k$, it will interior for any capital-level bigger than $k$. This follows from the concavity of the value function $V$. Hence, there is a threshold level of capital $k^*$, such that the constraint will be binding if and only if $k < k^*$.

Consider $k \geq k^*$. On this set, the solution is equal to the one given in the original solution, which neglected the constraint. Hence, for $k \geq k^*$, the value function is still given by

$$V(k) = \psi_0 + \psi_1 k + \psi_2 k^2,$$

where the coefficients $(\psi_0, \psi_1, \psi_2)$ are given in (S6.11), (S6.12) and (S6.16).

Now consider $k < k^*$. On this set, the constraint is binding by construction and the policy functions for capital and consumption are given by

$$k'(k) = Ak \text{ and } c(k) = 0.$$

Hence, the correct policy function for capital reads

$$k'(k) = \begin{cases} Ak & \text{if } k < k^* \\ \frac{1}{A\beta}k + \frac{\beta A - 1}{aA\beta(A-1)} & \text{if } k \geq k^* \end{cases},$$

and the policy function for consumption reads

$$c(k) = \begin{cases} 0 & \text{if } k < k^* \\ \frac{A^2\beta - 1}{A\beta}k - \frac{\beta A - 1}{aA\beta(A-1)} & \text{if } k \geq k^* \end{cases}.$$

At the threshold $k^*$, the multiplier $\lambda(k^*)$ is equal to zero at $c(k) = 0$. Hence, $k^*$ is defined by

$$\frac{A^2\beta - 1}{A\beta}k^* - \frac{\beta A - 1}{aA\beta(A-1)} = 0.$$

Solving this yields

$$k^* = \frac{\beta A - 1}{a(A-1)(A^2\beta - 1)},$$

which is positive because we assumed that $A\beta > 1$.

To solve for the value function for $k < k^*$, note that

$$\begin{aligned} V(k) &= Ak - k'(k) - \frac{a}{2}(Ak - k'(k))^2 + \beta V(k'(k)) \\ &= \beta V(Ak). \end{aligned}$$

This directly implies that $V(0) = 0$, which is intuitive, because consumption will be equal to zero in all periods. Additionally, $V(k^*)$ is known so that we can construct $V(k)$ for $k < k^*$. In particular, define

$$n(k) = \left\{n | A^n k \geq k^* \text{ and } A^{n-1}k < k^*\right\},$$

i.e. $n(k)$ is the number of periods such that starting at $k$, you reach at least $k^*$ without ever consuming. Then

$$V(k) = \beta^{n(k)} V\left(A^{n(k)}k\right),$$

where $A^{n(k)}k \geq k^*$ by construction and hence

$$V\left(A^{n(k)}k\right) = \psi_0 + \psi_1 A^{n(k)}k + \psi_2\left(A^{n(k)}k\right)^2.$$

# Chapter 14: Models of Schumpeterian Growth

## Exercise 14.7

There are two mistakes in this exercises. First of all, equation (S14.10) should read

$$\frac{\lambda^{\zeta_3}\eta}{q^{\zeta_3}} \frac{\beta q^{\zeta_2} q^{(\zeta_2 - \zeta_3)/\beta} L}{r^* + z^*} = 1.$$

This follows directly from (S14.9) but the original solution misses the $\zeta_3$ as the exponent of $\lambda$. This implies that (S14.12) should read

$$r^* + z^* = \lambda^{\zeta_3} \eta \beta L$$

and that the expressions for $z^*, r^*$ and $g^*$ in (S14.19) should be adapted accordingly.

The second mistake concerns the expression of aggregate expenditure on intermediate inputs $X(t)$. According to the exercise, $X(t)$ is given by

$$X(t) = (1-\beta) \int_0^1 q(\nu,t)^{\zeta_2} x(\nu,t|q)\, d\nu.$$

Substituting the expression for $x(\nu,t|q)$ (see (S14.6)) yields

$$\begin{aligned} X(t) &= (1-\beta) \int_0^1 q(\nu,t)^{\zeta_2} q(\nu,t)^{(\zeta_1-\zeta_2)/\beta}\, d\nu L \\ &= (1-\beta) \int_0^1 q(\nu,t)^{\zeta_2+(\zeta_1-\zeta_2)/\beta}\, d\nu L. \end{aligned}$$

Hence, the growth rate of $X(t)$ is given by

$$g_X = z^* \left( \lambda^{\zeta_2+(\zeta_1-\zeta_2)/\beta} - 1 \right),$$

which differs from the expression given in (S14.15). In particular, note that

$$\begin{aligned} \zeta_2 + \frac{(\zeta_1-\zeta_2)}{\beta} &= \zeta_1 - (\zeta_1-\zeta_2) + \frac{(\zeta_1-\zeta_2)}{\beta} \\ &= \zeta_1 + \frac{(\zeta_1-\zeta_2)(1-\beta)}{\beta}, \end{aligned}$$

so that $g_X = g_Y$ as can be seen from (S14.14). This implies that the restriction given in (S14.17) is not necessary, i.e. given restriction (S14.11), $X(t)$ and $Y(t)$ grow at the same rate regardless of $\zeta_1$ and $\zeta_2$. The implication that $\zeta_1 = \zeta_2 = \zeta_3$, which is derived in the original solution, is therefore also incorrect.